heat simulation of a coffee mug with NGSolve

Scientific Computing and FEM

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We investigate a heat conductivity problem for an idealized ceramic mug in 2D. We will consider the heating-up of the mug as well as the cooling down of a coffee filled in a preheated mug.

1 Mathematical and physical aspects

We consider the time dependent heat conductivity problem with Robin boundary conditions, no internal heat source and an initial condition.

$$
\begin{aligned}
c(x)\frac{\partial u(x,t)}{\partial t} - \nabla_x^T(\lambda(x)\nabla_x u(x,t)) = f(x,t) = 0 \qquad (x,t) \in \Omega \times (0,T), \\
-\lambda(x)\frac{\partial u(x,t)}{\partial \vec{n}} = \alpha(x)(u(x,t) - u_{out}) \qquad (x,t) \in \partial\Omega \times (0,T), \\
u(x,0) = u_0(x) \qquad x \in \overline{\Omega} = \overline{\Omega}^{wall} \cup \overline{\Omega}^{fluid} \cup \overline{\Omega}^{air}.
\end{aligned}
$$

For the solution we consider a spacial FEM (done with NGSolve) combined with a time-stepping scheme. So the following variational formulation results for this problem

$$\int_\Omega c\frac{\partial u}{\partial t} v dx + \int_\Omega \lambda \nabla u \nabla v dx + \int_{\partial\Omega} \alpha u v ds_x = \int_{\partial\Omega} \alpha u_{out} v ds_x.$$

So we end up with the following system for computing the solution $u(x,t)$

$$\left(\frac{1}{\tau}M + \theta A\right) u^{n+1} = \left(\frac{1}{\tau}M - (1-\theta)A\right) u^n + F,$$

where M represents the classical mass matrix, A the stiffness matrix (second and third integral), F the right hand side of the variational formulation and u^n is the solution of the actual time step. θ is a scaling parameter for the time-stepping method. We choose $\theta = 0.5$, the so called Crank-Nicolson scheme.

	ceramic	liquid	air
thermal conductivity λ (W/(m*K))	5	0.6	0.026
heat storage capacity c (J/(kg*K))	$1240 * 2.3 * 10^3$	$4.138 * 10^6$	$1.21 * 10^3$

Table 1: material coefficients for subdomains

The appropriate material coefficients c, λ, α differ with respect to the various subdomains and do not depend on time. For the specific values of λ, c see table 1. The heat transfer coefficient is set to $\alpha = 8$ (W/(m^{2}K)). Water and coffee have nearly the same material parameters, so those for liquid are valid for both.

First of all we need to generate the two dimensional mesh of our mug without handle. It consists of the three subdomains ceramic, liquid and air. The generated mesh with its subdomains is illustrated in graphic 1. Blue is ceramic, green stands for the liquid and red for the air part.

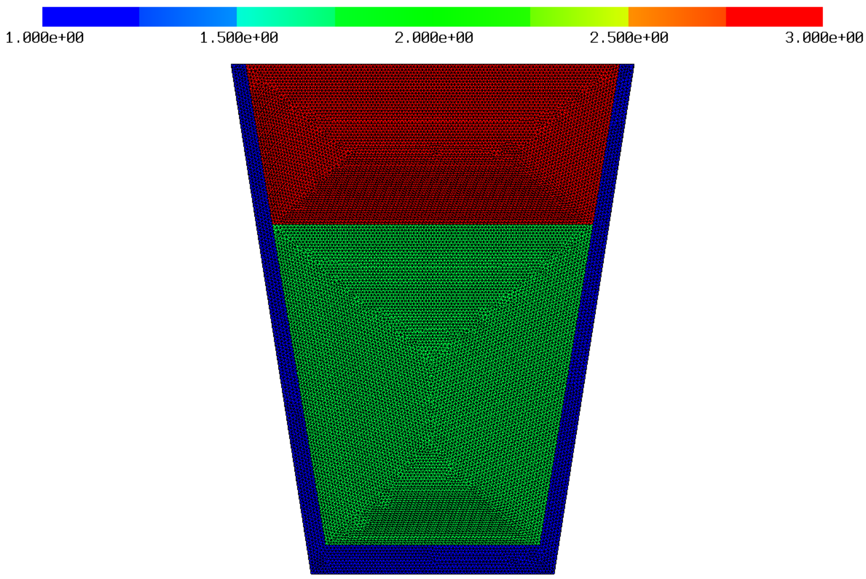

Figure 1: mesh with subdomains

2 Simulations

2.1 Preheating the cup - Simulation 1

We assume a cold mug which gets filled with hot water in order to preheat the ceramic. It should be investigated how long it takes until the mug reaches its optimal average temperature for keeping a coffee warm. The goal temperature would be 67 °C. However, with the given parameters this temperature cannot be achieved, since the mug already starts cooling before reaching 67 °C. The initial data can be seen in the table 2.

math		value (in °C)
$u_0^{ceramic}$	initial temperature of ceramic	18
u_0^{liquid}	initial temperature of liquid	80
u_0^{air}	initial temperature of air	18
u_{out}	outside temperature	18

Table 2: initial data for simulation 1

After 451 seconds the mug reaches its maximal average temperature of 51.478 °C. In figure 2 you can see the initial heat distribution. Figure 3 shows the heat distribution after 451 seconds, when reaching the maximal temperature. And the figure 4 presents the temporal evolution of the ceramic's average temperature.

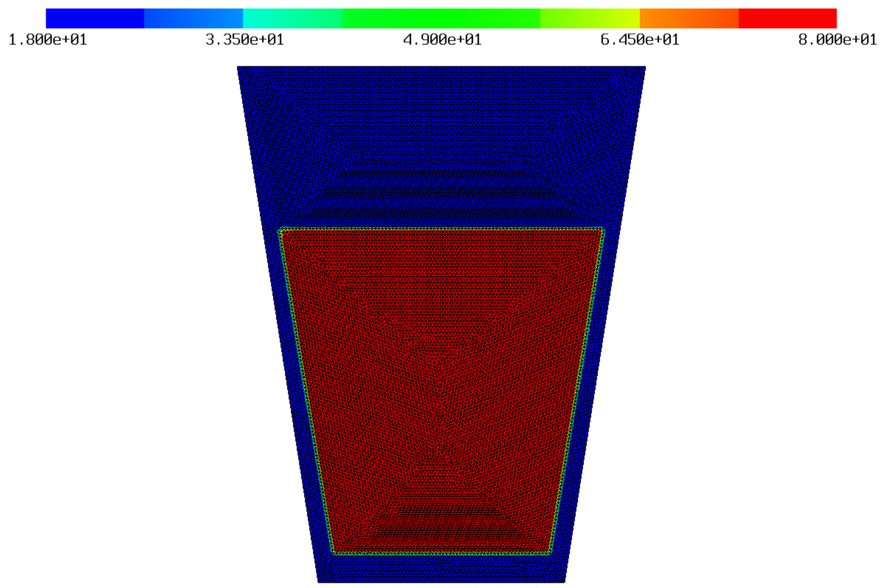

Figure 2: initial heat distribution for simulation 1

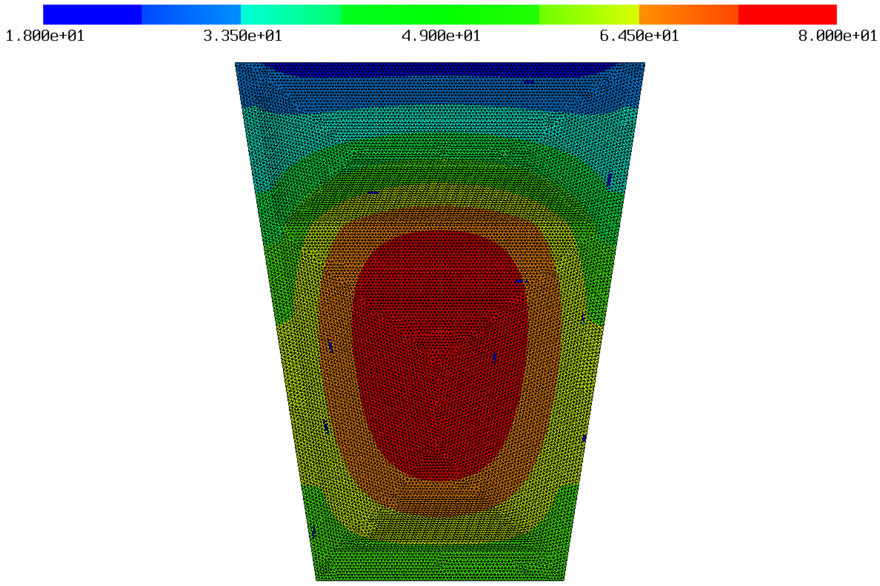

Figure 3: heat distribution after 451 sec (simulation 1)

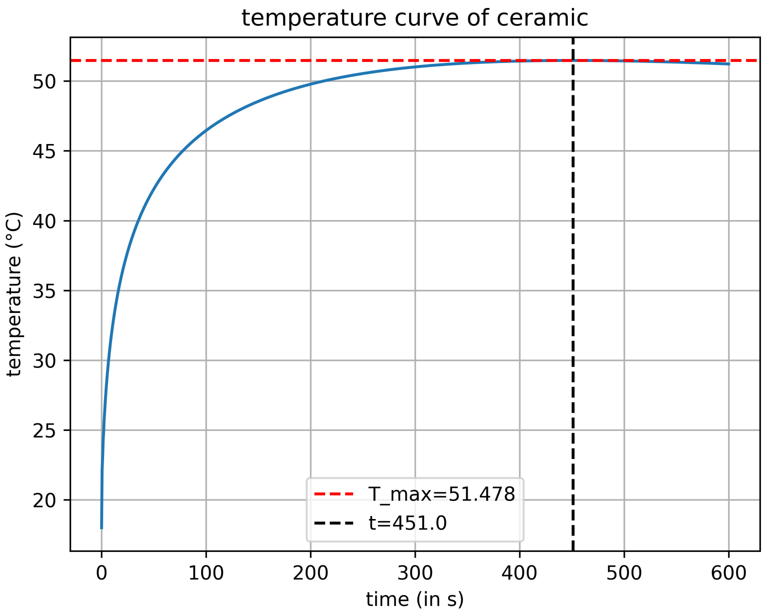

Figure 4: average ceramic temperature over time (simulation 1)

2.2 Cooling down of coffee - Simulation 2

We take the preheated mug from the previous simulation 1 and replace the water inside with hot coffee. The question now is how long the coffee remains warm above 50 °C. The initial data for this simulation can be seen in table 3.

math		value (in °C)
$u_0^{ceramic}$	initial temperature of ceramic	from simulation 1
u_0^{liquid}	initial temperature of liquid	85
u_0^{air}	initial temperature of air	from simulation 1
u_{out}	outside temperature	18

Table 3: initial data for simulation 2

After 3597.5 seconds (approximately one hour) the average temperature of the coffee drops below 50 °C and we stop the simulation. Figure 5 shows the heat distribution at the the beginning of simulation 2. So it is the result from simulation 1, where the water got replaced by coffee. In figure 6 one can see the heat distribution at that time when the average coffee temperature drops under 50 °C. Figure 7 illustrates the decreasing coffee temperature over time.

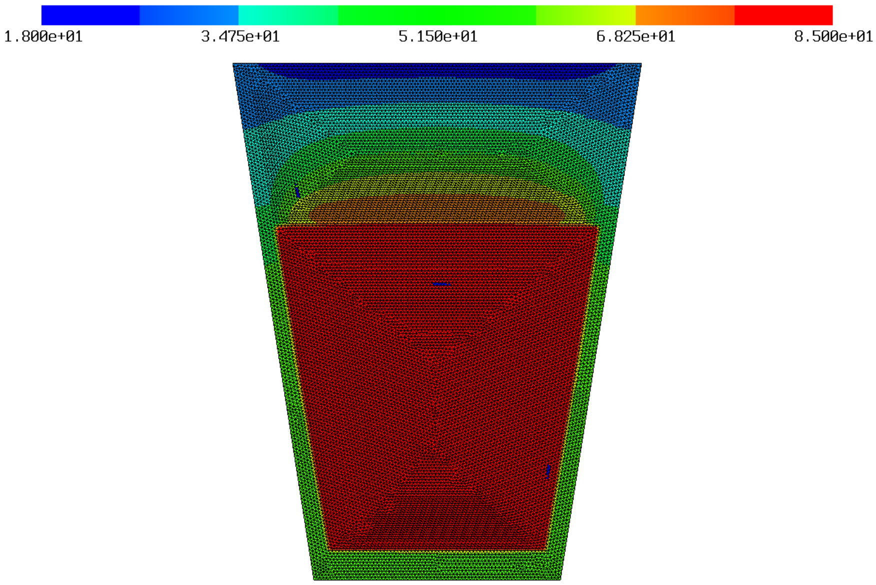

Figure 5: initial heat distribution for simulation 2

Videos of both simulations can be found on github.

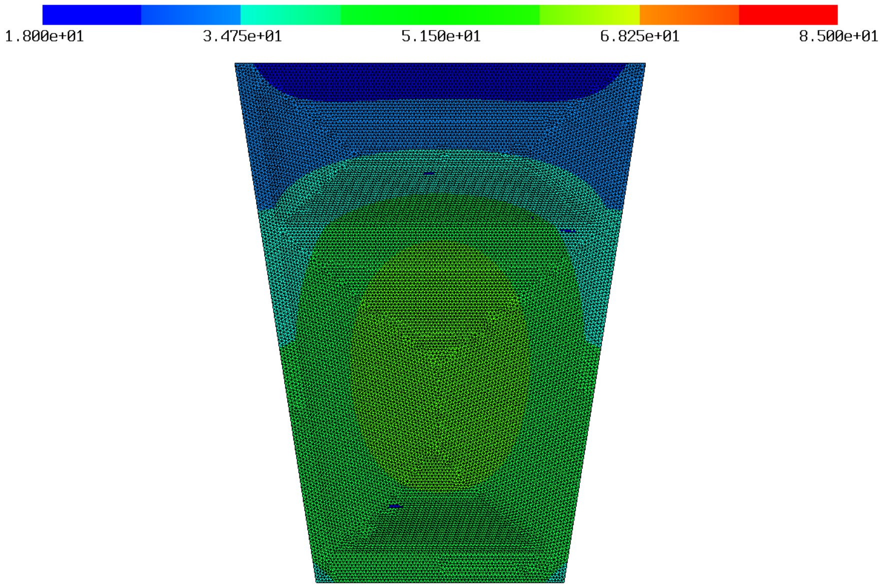

Figure 6: heat distribution after 3597.5 sec (simulation 2)

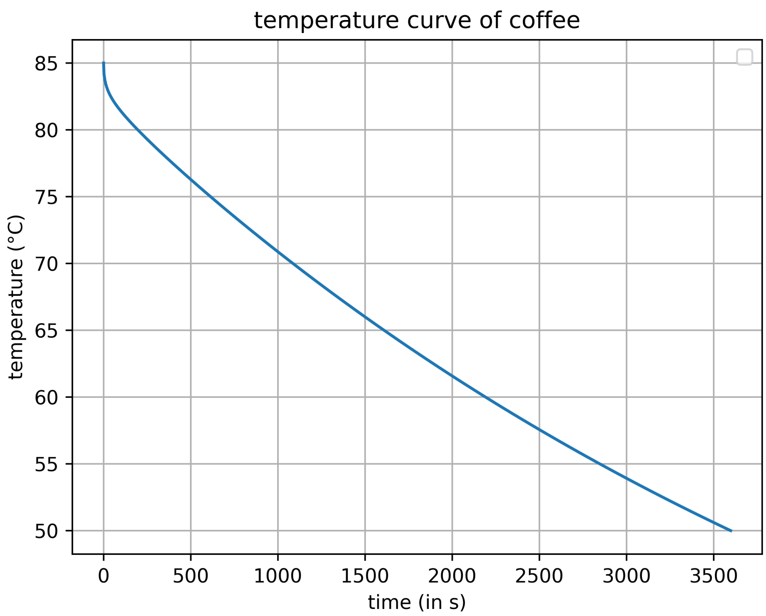

Figure 7: average coffee temperature over time (simulation 2)