High Performance Computing (Master) in WS25

Exercise 2: PDEs modelling and analysis.

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(A) Model the stationary heat equation in the 1D domain $\Omega = (0,1)$ without convection and without advection. [3 pts]
Assume Dirichlet boundary conditions $u(0) = 0$ and $u(1) = 1$ for the unknown solution $u(x)$. Assume a constant heating $f(x) := c$ and a constant heat conductivity $\lambda(x)$ in the domain.

- Which function spaces are required for u, f and λ in that PDE?
- The solution can be expressed as $u(x) = u^H(x) + u^f(x)$ with
  - $u^H(x)$ as *homogeneous* solution of the PDE with $f(x) := 0$.
  - $u^f(x)$ as solution of the PDE with zero Dirichlet b.c. $u(0) = u(1) = 0$.

  Formulate both PDEs, determine $u^H(x)$ and $u^f(x)$ analytically and check correctness of $u(x)$ in the original PDE.

(B) Model the stationary heat equation in the 1D domain $\Omega = (0,1)$ without convection and without advection. [2 pts]
Assume Dirichlet boundary conditions $u(0) = 0$ and $u(1) = 1$ for the unknown solution $u(x)$. There are no internal source $f(x) := 0$ and we have heterogeneous material $\lambda(x) = \begin{cases} 1 & x \in (0, 0.5) \\ 10 & x \in (0.5, 1) \end{cases}$, i.e., the heat conductivity changes.

- Use the strong formulation of the PDE with interface conditions.
- Which function spaces are required for u, f and λ in that PDE?
- Determine the solution u(x) analytically.

(C) Solve the Péclet problem [2 pts]

$$\begin{aligned} -u''(x) + pu'(x) &= 0 \qquad x \in (0,1) \\ u(0) &= 0 \\ u(1) &= 1 \end{aligned}$$

analytically with a constant $p \in \mathbb{R}$.

- Hint: Substitute $z(x) = u'(x)$, solve the ODE and incorporate the b.c..
- Visualize (Matlab, python, ...) your solution for an increasing $p > 0$ as well as for a decreasing $p < 0$.