```
    1: #undef CUTHILL_MCKEE
    2: // see:   http://llvm.org/docs/CodingStandards.html#include-style
    3: #include "vdop.h"
    4: #ifdef CUTHILL_MCKEE
    5: #include "cuthill_mckee_ordering.h"
    6: #endif
    7: #include "geom.h"
    8:
    9: #include <algorithm>
   10: #include <array>
   11: #include <cassert>
   12: #include <cmath>
   13: #include <ctime>                    // contains clock()
   14: #include <fstream>
   15: #include <iostream>
   16: #include <list>
   17: #include <string>
   18: #include <vector>
   19:
   20: using namespace std;
   21:
   22: Mesh::Mesh(int ndim, int nvert_e, int ndof_e, int nedge_e)
   23:     : _nelem(0), _nvert_e(nvert_e), _ndof_e(ndof_e), _nnode(0), _ndim(ndim), _ia(0),
 _xc(0),
   24:       _bedges(0), _sdedges(0),
   25:       _nedge(0), _nedge_e(nedge_e), _edges(0), _ea(), _ebedges(),
   26:       _dummy(0)
   27: {
   28: }
   29:
   30: Mesh::~Mesh()
   31: {}
   32:
   33: void Mesh::SetValues(std::vector<double> &v, const function<double(double, double)>
&func) const
   34: {
   35:     int const nnode = Nnodes();            // number of vertices in mesh
   36:     assert( nnode == static_cast<int>(v.size()) );
   37:     for (int k = 0; k < nnode; ++k)
   38:     {
   39:         v[k] = func( _xc[2 * k], _xc[2 * k + 1] );
   40:     }
   41: }
   42:
   43: void Mesh::SetBoundaryValues(vector<double> &v, const function<double(double, double
)> &func) const
   44: {
   45:     auto const idx = Index_BoundaryNodes();
   46:     for (size_t ik = 0; ik < idx.size(); ++ik)
   47:     {
   48:         const int k = idx[ik];
   49:         v[k] = func( _xc[2 * k], _xc[2 * k + 1] );
   50:     }
   51: }
   52:
   53: void Mesh::SetDirchletValues(vector<double> &v, const function<double(double, double
)> &func) const
   54: {
   55:     auto const idx = Index_DirichletNodes();
   56:     for (size_t ik = 0; ik < idx.size(); ++ik)
   57:     {
   58:         const int k = idx[ik];
   59:         v[k] = func( _xc[2 * k], _xc[2 * k + 1] );
   60:     }
   61: }
   62:
   63:
   64:
```

```
   65: void Mesh::Debug() const
   66: {
   67:     cout << "\n ############### Debug  M E S H  ###################\n";
   68:     cout << "\n ..............    Coordinates       ..................\n";
   69:     for (int k = 0; k < _nnode; ++k)
   70:     {
   71:         cout << k << " : " ;
   72:         for (int i = 0; i < _ndof_e; ++i )
   73:         {
   74:             cout << _xc[2*k+i] << "  ";
   75:         }
   76:         cout << endl;
   77:     }
   78:     cout << "\n ..............    Elements        ..................\n";
   79:     for (int k = 0; k < _nelem; ++k)
   80:     {
   81:         cout << k << " : ";
   82:         for (int i = 0; i < _ndof_e; ++i )
   83:             cout << _ia[_ndof_e * k + i] << "  ";
   84:         cout << endl;
   85:     }
   86:     cout << "\n ..............    Boundary (vertices)    .................\n";
   87:     cout << " _bedges : " << _bedges << endl;
   88:     return;
   89: }
   90:
   91: void Mesh::DebugEdgeBased() const
   92: {
   93:     cout << "\n ############### Debug  M E S H  (edge based) ###################\n";
   94:     cout << "\n ..............    Coordinates       ..................\n";
   95:     for (int k = 0; k < _nnode; ++k)
   96:     {
   97:         cout << k << " : " << _xc[2 * k] << "  " << _xc[2 * k + 1] << endl;
   98:     }
   99:
  100:     cout << "\n ..............    edges        ..................\n";
  101:     for (int k = 0; k < _nedge; ++k)
  102:     {
  103:         cout << k << " : ";
  104:         for (int i = 0; i < 2; ++i )
  105:             cout << _edges[2 * k + i] << "  ";
  106:         cout << endl;
  107:     }
  108:
  109:     cout << "\n ..............    Elements (edges)    .................\n";
  110:     assert(_nedge_e * _nelem == static_cast<int>(_ea.size()) );
  111:     for (int k = 0; k < _nelem; ++k)
  112:     {
  113:         cout << k << " : ";
  114:         for (int i = 0; i < _nedge_e; ++i )
  115:             cout << _ea[_nedge_e * k + i] << "  ";
  116:         cout << endl;
  117:     }
  118:     cout << "\n ..............    Boundary (edges)    .................\n";
  119:     cout << " _ebedges : " << _ebedges << endl;
  120:
  121:     return;
  122: }
  123:
  124: void Mesh::Write_ascii_matlab(std::string const &fname, std::vector<double> const &v
) const
  125: {
  126:     assert(Nnodes() ==  static_cast<int>(v.size()));  // fits vector length to mesh
information?
  127:
  128:     ofstream fout(fname);                                // open file ASCII mode
  129:     if ( !fout.is_open() )
  130:     {
```

```
  131:         cout << "\nFile " << fname << " has not been opened.\n\n" ;
  132:         assert( fout.is_open() && "File not opened."  );
  133:     }
  134:
  135:     string const DELIMETER(" ");    // define the same delimiter as in matlab/ascii_
read*.m
  136:     int const    OFFSET(1);         // convert C-indexing to matlab
  137:
  138:     // Write data: #nodes, #space dimensions, #elements, #vertices per element
  139:     fout << Nnodes() << DELIMETER << Ndims() << DELIMETER << Nelems() << DELIMETER <
< NverticesElements() << endl;
  140:
  141:     // Write coordinates: x_k, y_k   in separate lines
  142:     assert( Nnodes()*Ndims() ==  static_cast<int>(_xc.size()));
  143:     for (int k = 0, kj = 0; k < Nnodes(); ++k)
  144:     {
  145:         for (int j = 0; j < Ndims(); ++j, ++kj)
  146:         {
  147:             fout << _xc[kj] << DELIMETER;
  148:         }
  149:         fout << endl;
  150:     }
  151:
  152:     // Write connectivity: ia_k,0, ia_k,1 etc  in separate lines
  153:     assert( Nelems()*NverticesElements() ==  static_cast<int>(_ia.size()));
  154:     for (int k = 0, kj = 0; k < Nelems(); ++k)
  155:     {
  156:         for (int j = 0; j < NverticesElements(); ++j, ++kj)
  157:         {
  158:             fout << _ia[kj] + OFFSET << DELIMETER;     // C to matlab
  159:         }
  160:         fout << endl;
  161:     }
  162:
  163:     // Write vector
  164:     for (int k = 0; k < Nnodes(); ++k)
  165:     {
  166:         fout << v[k] << endl;
  167:     }
  168:
  169:     fout.close();
  170:     return;
  171: }
  172:
  173:
  174: void Mesh::Export_scicomp(std::string const &basename) const
  175: {
  176:     //assert(Nnodes() ==  static_cast<int>(v.size()));  // fits vector length to mes
h information?
  177:     string const DELIMETER(" ");    // define the same delimiter as in matlab/ascii_
read*.m
  178:     int const    OFFSET(0);
  179:     {
  180:         // Write coordinates into scicomp-file
  181:         string fname(basename + "_coords.txt");
  182:         ofstream fout(fname);                                  // open file ASCII mode
  183:         if ( !fout.is_open() )
  184:         {
  185:             cout << "\nFile " << fname << " has not been opened.\n\n" ;
  186:             assert( fout.is_open() && "File not opened."  );
  187:         }
  188:
  189:         fout << Nnodes() << endl;
  190:         // Write coordinates: x_k, y_k   in separate lines
  191:         assert( Nnodes()*Ndims() ==  static_cast<int>(_xc.size()));
  192:         for (int k = 0, kj = 0; k < Nnodes(); ++k)
  193:         {
  194:             for (int j = 0; j < Ndims(); ++j, ++kj)
```

```
  195:             {
  196:                 fout << _xc[kj] << DELIMETER;
  197:             }
  198:             fout << endl;
  199:         }
  200:         fout.close();
  201:
  202:     }
  203:
  204:     {
  205:         // Write elements into scicomp-file
  206:         string fname(basename + "_elements.txt");
  207:         ofstream fout(fname);                                   // open file ASCII mode
  208:         if ( !fout.is_open() )
  209:         {
  210:             cout << "\nFile " << fname << " has not been opened.\n\n" ;
  211:             assert( fout.is_open() && "File not opened."  );
  212:         }
  213:
  214:         fout << Nelems() << endl;
  215:
  216:         // Write connectivity: ia_k,0, ia_k,1 etc  in separate lines
  217:         assert( Nelems()*NverticesElements() ==  static_cast<int>(_ia.size()));
  218:         for (int k = 0, kj = 0; k < Nelems(); ++k)
  219:         {
  220:             for (int j = 0; j < NverticesElements(); ++j, ++kj)
  221:             {
  222:                 fout << _ia[kj] + OFFSET << DELIMETER;     // C to matlab
  223:             }
  224:             fout << endl;
  225:         }
  226:         fout.close();
  227:     }
  228:
  229:     return;
  230: }
  231:
  232: /*
  233:   manjaro> matlab
  234:   MATLAB is selecting SOFTWARE OPENGL rendering.
  235:   /usr/local/MATLAB/R2019a/bin/glnxa64/MATLAB: error while loading shared libraries:
 libcrypt.so.1: cannot open shared object file: No such file or directory
  236:
  237:   SOLUTION: sudo pacman -S libxcrypt-compat     + reboot
  238: */
  239:
  240: void Mesh::Visualize(vector<double> const &v) const
  241: {
  242:     // define external command
  243:     // const string exec_m("matlab -nosplash < visualize_results.m");
  // Matlab
  244:     // const string exec_m("octave --no-window-system --no-gui visualize_results.m")
; // Octave until version 6.3
  245:
  246:     // for visualization I had to type in terminal:
  247:     // export LIBGL_ALWAYS_SOFTWARE=1
  248:     const string exec_m("octave --no-gui --eval visualize_results"); // Octave since
 version 6.4
  249:
  250:     // const string exec_m("flatpak run org.octave.Octave visualize_results.m");
  // Octave (flatpak): desktop GH
  251:
  252:     const string fname("uv.txt");
  253:     Write_ascii_matlab(fname, v);
  254:
  255:     int ierror = system(exec_m.c_str());                                  // call ext
ernal command
  256:
```

```
  257:     if (ierror != 0)
  258:     {
  259:         cout << endl << "Check path to Matlab/octave on your system" << endl;
  260:     }
  261:     cout << endl;
  262:     return;
  263: }
  264:
  265: std::vector<int> Mesh::Index_DirichletNodes() const
  266: {
  267:     //vector<int> idx(_bedges);                                 // copy
  268:     // 2020-01-08
  269:     //                  copy only the Dirichlet boundary nodes, not all boundary node
s.
  270:     vector<int> idx(_bedges.size());
  271:     size_t cnt=0;
  272:     for (size_t kb = 0; kb < _bedges.size(); kb+=2 )
  273:     {
  274:         if (_sdedges.at(kb) <0 || _sdedges.at(kb+1) <0) // one neighboring subdomain
 is negativ
  275:         {
  276:             idx[cnt] = _bedges[kb];
  277:             ++cnt;
  278:             idx[cnt] = _bedges[kb+1];
  279:             ++cnt;
  280:         }
  281:     }
  282:     idx.resize(cnt);
  283:
  284:     sort(idx.begin(), idx.end());                             // sort
  285:     idx.erase( unique(idx.begin(), idx.end()), idx.end() ); // remove duplicate data
  286:
  287:     return idx;
  288: }
  289:
  290: // 2020-01-08
  291: std::vector<int> Mesh::Index_BoundaryNodes() const
  292: {
  293:     vector<int> idx(_bedges);                                  // copy
  294:
  295:     sort(idx.begin(), idx.end());                             // sort
  296:     idx.erase( unique(idx.begin(), idx.end()), idx.end() ); // remove duplicate data
  297:
  298:     return idx;
  299: }
  300:
  301: // GH
  302: //  only correct for simplices
  303: void Mesh::DeriveEdgeFromVertexBased_fast_2()
  304: {
  305:     assert(NedgesElements() == 3);
  306:     assert(NverticesElements() == 3);   // 3 vertices, 3 edges per element are assum
ed
  307:
  308:     // Store indices of all elements connected to a vertex
  309:     vector<vector<int>> vertex2elems(_nnode, vector<int>(0));
  310:     for (int k = 0; k < Nelems(); ++k)
  311:     {
  312:         for (int i = 0; i < 3; ++i)
  313:         {
  314:             vertex2elems[_ia[3 * k + i]].push_back(k);
  315:         }
  316:     }
  317:     size_t max_neigh = 0;             // maximal number of elements per vertex
  318:     for (auto const &v : vertex2elems)
  319:     {
  320:         max_neigh = max(max_neigh, v.size());
  321:     }
```

```
  322:     //cout << endl << vertex2elems << endl;
  323:
  324:     // assign edges to elements
  325:     _ea.clear();                        // old data still in _ea without clear()
  326:     _ea.resize(NedgesElements()*Nelems(), -1);
  327:     // Derive the edges
  328:     _edges.clear();
  329:     _nedge = 0;
  330:
  331:     // convert also boundary edges
  332:     unsigned int mbc(_bedges.size() / 2);     // number of boundary edges
  333:     _ebedges.clear();
  334:     _ebedges.resize(mbc, -1);
  335:     vector<bool> bdir(_nnode, false);         // vector indicating boundary nodes
  336:     //for (size_t kb = 0; kb < _bedges.size(); ++kb )
  337:     //{
  338:         //bdir.at(_bedges[kb]) = true;
  339:     //}
  340:     // 2020-01-08
  341:     // GH ToDo: Selection of Dirichlet edges/nodes is still wrong
  342:     //          Problem: _bedges is used externally instead of the local correct bdi
r.
  343:     for (size_t kb = 0; kb < _bedges.size(); kb+=2 )
  344:     {
  345:         bool const booldir = _sdedges.at(kb) <0 || _sdedges.at(kb+1) <0; // one neig
hboring subdomain is negativ
  346:         bdir.at(_bedges[kb])   = booldir;
  347:         bdir.at(_bedges[kb+1]) = booldir;
  348:     }
  349:
  350:     vector<int> vert_visited;             // already visisted neighboring vertices o
f k
  351:     vert_visited.reserve(max_neigh);      // avoids multiple (re-)allocations
  352:     for (int k = 0; k < _nnode; ++k)              // vertex k
  353:     {
  354:         vert_visited.clear();
  355:         auto const &elems = vertex2elems[k];      // element neighborhood
  356:         int kedges = static_cast<int>(_edges.size()) / 2; // #edges before vertex k
is investigated
  357:         //cout << elems << endl;
  358: // GH: problem, shared edges appear twice.
  359:         int nneigh = elems.size();
  360:         for (int ne = 0; ne < nneigh; ++ne)       // iterate through neighborhood
  361:         {
  362:             int e = elems[ne];                    // neighboring element e
  363:             //cout << "e = " << e << endl;
  364:             for (int i = 3 * e + 0; i < 3 * e + _nvert_e; ++i)   // vertices of elem
ent e
  365:             {
  366:                 int const vert = _ia[i];
  367:                 //cout << "vert: " << vert << "  "<< k << endl;
  368:                 if ( vert > k )
  369:                 {
  370:                     int ke = -1;
  371:                     auto const iv = find(vert_visited.cbegin(), vert_visited.cend(),
 vert);
  372:                     if (iv == vert_visited.cend())   // vertex not yet visited
  373:                     {
  374:                         vert_visited.push_back(vert);  // now, vertex vert is visite
d
  375:                         _edges.push_back(k);           // add the new edge k->vert
  376:                         _edges.push_back(vert);
  377:
  378:                         ke = _nedge;
  379:                         ++_nedge;
  380:                         // Is edge ke also a boundary edge?
  381:                         if (bdir[k] && bdir[vert])
  382:                         {
```

```
  383:                                 size_t kb = 0;
  384:                                 while (kb < _bedges.size() && (!( (_bedges[kb] == k && _
bedges[kb + 1] == vert) || (_bedges[kb] == vert && _bedges[kb + 1] == k) )) )
  385:                                 {
  386:                                     kb += 2;
  387:                                 }
  388:                                 if (kb < _bedges.size())
  389:                                 {
  390:                                     _ebedges[kb / 2] = ke;
  391:                                 }
  392:                             }
  393:                         }
  394:                         else
  395:                         {
  396:                             int offset = iv - vert_visited.cbegin();
  397:                             ke = kedges + offset;
  398:                         }
  399:                         // assign that edge to the edges based connectivity of element e
  400:                         auto ip = find_if(_ea.begin() + 3 * e, _ea.begin() + 3 * (e + 1)
,
  401:                                           [] (int v) -> bool {return v < 0;} );
  402:                         //cout << ip-_ea.begin()+3*e << "  " << *ip << endl;
  403:                         assert(ip != _ea.cbegin() + 3 * (e + 1)); // data error !
  404:                         *ip = ke;
  405:                     }
  406:                 }
  407:             }
  408:         }
  409:
  410:         assert( Mesh::Check_array_dimensions() );
  411:         return;
  412: }
  413: // HG
  414:
  415: // GH
  416: //  only correct for simplices
  417: void Mesh::DeriveEdgeFromVertexBased_fast()
  418: {
  419:     assert(NedgesElements() == 3);
  420:     assert(NverticesElements() == 3);   // 3 vertices, 3 edges per element are assum
ed
  421:
  422:     // Store indices of all elements connected to a vertex
  423:     vector<vector<int>> vertex2elems(_nnode, vector<int>(0));
  424:     for (int k = 0; k < Nelems(); ++k)
  425:     {
  426:         for (int i = 0; i < 3; ++i)
  427:         {
  428:             vertex2elems[_ia[3 * k + i]].push_back(k);
  429:         }
  430:     }
  431:     size_t max_neigh = 0;             // maximal number of elements per vertex
  432:     for (auto const &v : vertex2elems)
  433:     {
  434:         max_neigh = max(max_neigh, v.size());
  435:     }
  436:     //cout << endl << vertex2elems << endl;
  437:
  438:     // assign edges to elements
  439:     _ea.clear();                      // old data still in _ea without clear()
  440:     _ea.resize(NedgesElements()*Nelems(), -1);
  441:     // Derive the edges
  442:     _edges.clear();
  443:     _nedge = 0;
  444:     vector<int> vert_visited;             // already visisted neighboring vertices o
f k
  445:     vert_visited.reserve(max_neigh);      // avoids multiple (re-)allocations
  446:     for (int k = 0; k < _nnode; ++k)              // vertex k
```

```
  447:     {
  448:         vert_visited.clear();
  449:         auto const &elems = vertex2elems[k];      // element neighborhood
  450:         int kedges = static_cast<int>(_edges.size()) / 2; // #edges before vertex k
is investigated
  451:         //cout << elems << endl;
  452: // GH: problem, shared edges appear twice.
  453:         int nneigh = elems.size();
  454:         for (int ne = 0; ne < nneigh; ++ne)       // iterate through neighborhood
  455:         {
  456:             int e = elems[ne];                    // neighboring element e
  457:             //cout << "e = " << e << endl;
  458:             for (int i = 3 * e + 0; i < 3 * e + _nvert_e; ++i)   // vertices of elem
ent e
  459:             {
  460:                 int const vert = _ia[i];
  461:                 //cout << "vert: " << vert << "  "<< k << endl;
  462:                 if ( vert > k )
  463:                 {
  464:                     int ke = -1;
  465:                     auto const iv = find(vert_visited.cbegin(), vert_visited.cend(),
 vert);
  466:                     if (iv == vert_visited.cend())   // vertex not yet visited
  467:                     {
  468:                         vert_visited.push_back(vert);  // now, vertex vert is visite
d
  469:                         _edges.push_back(k);           // add the new edge k->vert
  470:                         _edges.push_back(vert);
  471:
  472:                         ke = _nedge;
  473:                         ++_nedge;
  474:                     }
  475:                     else
  476:                     {
  477:                         int offset = iv - vert_visited.cbegin();
  478:                         ke = kedges + offset;
  479:                     }
  480:                     // assign that edge to the edges based connectivity of element e
  481:                     auto ip = find_if(_ea.begin() + 3 * e, _ea.begin() + 3 * (e + 1)
,
  482:                                       [] (int v) -> bool {return v < 0;} );
  483:                     //cout << ip-_ea.begin()+3*e << "  " << *ip << endl;
  484:                     assert(ip != _ea.cbegin() + 3 * (e + 1)); // data error !
  485:                     *ip = ke;
  486:                 }
  487:             }
  488:         }
  489:     }
  490:
  491:     // convert also boundary edges
  492:     unsigned int mbc(_bedges.size() / 2);     // number of boundary edges
  493:     _ebedges.clear();
  494:     _ebedges.resize(mbc, -1);
  495:     for (unsigned int kb = 0; kb < mbc; ++kb)
  496:     {
  497:         int const v1 = min(_bedges[2 * kb], _bedges[2 * kb + 1]); // vertices
  498:         int const v2 = max(_bedges[2 * kb], _bedges[2 * kb + 1]);
  499:
  500:         size_t e = 0;
  501:         //   ascending vertex indices for each edge e in _edges
  502:         while (e < _edges.size() && (_edges[e] != v1 || _edges[e + 1] != v2) )
  503:         {
  504:             e += 2;                               // next edge
  505:         }
  506:         assert(e < _edges.size());                // error: no edge found
  507:         _ebedges[kb] = e / 2;                     // index of edge
  508:     }
  509:
```

```
  510:
  511:     assert( Mesh::Check_array_dimensions() );
  512:     return;
  513: }
  514: // HG
  515:
  516:
  517: #include <utility>              // pair
  518:
  519: void Mesh::DeriveEdgeFromVertexBased_slow()
  520: {
  521:     assert(NedgesElements() == 3);
  522:     assert(NverticesElements() == 3);   // 3 vertices, 3 edges per element are assum
ed
  523:
  524:     _ea.resize(NedgesElements()*Nelems());
  525:     vector< pair<int, int> >  edges(0);
  526:     int nedges = 0;
  527:
  528:     for (int k = 0; k < Nelems(); ++k)
  529:     {
  530:         array < int, 3 + 1 > ivert{{ _ia[3 * k], _ia[3 * k + 1], _ia[3 * k + 2], _ia
[3 * k] }};
  531:
  532:         for (int i = 0; i < 3; ++i)
  533:         {
  534:             pair<int, int> e2;          // this edge
  535:             if (ivert[i] < ivert[i + 1])   // guarantee ascending order
  536:             {
  537:                 e2 = make_pair(ivert[i], ivert[i + 1]);
  538:             }
  539:             else
  540:             {
  541:                 e2 = make_pair(ivert[i + 1], ivert[i]);
  542:             }
  543:
  544:             int eki(-1);                // global index of this edge
  545:             auto ip = find(edges.cbegin(), edges.cend(), e2);
  546:             if ( ip == edges.cend() )   // edge not found ==> add that edge
  547:             {
  548:                 //cout << "found edge\n";
  549:                 edges.push_back(e2);    // add the new edge
  550:                 eki = nedges;           // index of this new edge
  551:                 ++nedges;
  552:
  553:             }
  554:             else
  555:             {
  556:                 eki = ip - edges.cbegin(); // index of the edge found
  557:             }
  558:             _ea[3 * k + i] = eki;          // set edge index in edge based connectiv
ity
  559:         }
  560:     }
  561:
  562:     assert( nedges == static_cast<int>(edges.size()) );
  563:     _nedge = nedges;                    // set the member variable for number of edg
es
  564:     _edges.resize(2 * nedges);          // allocate memory for edge storage
  565:     for (int k = 0; k < nedges; ++k)
  566:     {
  567:         _edges[2 * k    ] = edges[k].first;
  568:         _edges[2 * k + 1] = edges[k].second;
  569:     }
  570:
  571:     // convert also boundary edges
  572:     unsigned int mbc(_bedges.size() / 2);     // number of boundary edges
  573:     //cout << "AA  " << mbc << endl;
```

```
  574:     _ebedges.resize(mbc);
  575:     for (unsigned int kb = 0; kb < mbc; ++kb)
  576:     {
  577:         const auto vv1 = make_pair(_bedges[2 * kb  ], _bedges[2 * kb + 1]); // both
  578:         const auto vv2 = make_pair(_bedges[2 * kb + 1], _bedges[2 * kb  ]); //  dire
ctions of edge
  579:         auto ip1 = find(edges.cbegin(), edges.cend(), vv1);
  580:         if (ip1 == edges.cend())
  581:         {
  582:             ip1 = find(edges.cbegin(), edges.cend(), vv2);
  583:             assert(ip1 != edges.cend());          // stop because inconsistency (bou
ndary edge has to be included in edges)
  584:         }
  585:         _ebedges[kb] = ip1 - edges.cbegin();      // index of edge
  586:     }
  587:
  588:     assert( Mesh::Check_array_dimensions() );
  589:     return;
  590: }
  591:
  592: void Mesh::DeriveVertexFromEdgeBased()
  593: {
  594:     assert(NedgesElements() == 3);
  595:     assert(NverticesElements() == 3);   // 3 vertices, 3 edges per element are assum
ed
  596:
  597:     _ia.resize(NedgesElements()*Nelems()); // NN
  598:
  599:     for (int k = 0; k < Nelems(); ++k)
  600:     {
  601:         //vector<int> ivert(6);           // indices of vertices
  602:         array<int, 6> ivert;          // indices of vertices
  603:         for (int j = 0; j < 3; ++j)     // local edges
  604:         {
  605:             int const iedg = _ea[3 * k + j]; // index of one edge in triangle
  606:             ivert[2 * j  ] = _edges[2 * iedg  ]; // first  vertex of edge
  607:             ivert[2 * j + 1] = _edges[2 * iedg + 1]; // second vertex of edge
  608:         }
  609:         sort(ivert.begin(), ivert.end()); // unique indices are needed
  610:         auto const ip = unique(ivert.begin(), ivert.end());
  611:         assert( ip - ivert.begin() == 3 );
  612:         for (int i = 0; i < 3; ++i)     // vertex based element connectivity
  613:         {
  614:             _ia[3 * k + i] = ivert[i];
  615:         }
  616:     }
  617:
  618:     // convert also boundary edges
  619:     unsigned int mbc(_ebedges.size());       // number of boundary edges
  620:     _bedges.resize(2 * mbc);
  621:     for (unsigned int k = 0; k < mbc; ++k)
  622:     {
  623:         const auto ke = _ebedges[k];        // edge index
  624:         _bedges[2 * k  ] = _edges[2 * ke  ];
  625:         _bedges[2 * k + 1] = _edges[2 * ke + 1];
  626:     }
  627:
  628:
  629:     return;
  630: }
  631:
  632: // Member Input: vertices of each element : _ia[_nelem*_nvert_e] stores as 1D array
  633: //     number of vertices per element     : _nvert_e
  634: //                global number of elements: _nelem
  635: //                global number of vertices: _nnode
  636: vector<vector<int>> Mesh::Node2NodeGraph_2() const
  637: {
  638:     vector<vector<int>> v2v(_nnode, vector<int>(0));    // stores the vertex to vert
```

```
ex connections
  639:
  640:     ////--------------
  641:     vector<int> cnt(_nnode,0);
  642:     for (size_t i = 0; i < _ia.size(); ++i)  ++cnt[_ia[i]]; // determine number of e
ntries per vertex
  643:     for (size_t k = 0; k < v2v.size(); ++k)
  644:     {
  645:         v2v[k].resize(_nvert_e * cnt[k]);              //    and allocate the memory
 for that vertex
  646:         cnt[k] = 0;
  647:     }
  648:     ////--------------
  649:
  650:     for (int e = 0; e < _nelem; ++e)
  651:     {
  652:         int const basis = e * _nvert_e;                  // start of vertex connecti
vity of element e
  653:         for (int k = 0; k < _nvert_e; ++k)
  654:         {
  655:             int const v = _ia[basis + k];
  656:             for (int l = 0; l < _nvert_e; ++l)
  657:             {
  658:                 v2v[v][cnt[v]] = _ia[basis + l];
  659:                 ++cnt[v];
  660:             }
  661:         }
  662:     }
  663:     // finally  cnt[v]==v2v[v].size()  has to hold for all v!
  664:
  665:     // guarantee unique, ascending sorted entries per vertex
  666:     for (size_t v = 0; v < v2v.size(); ++v)
  667:     {
  668:         sort(v2v[v].begin(), v2v[v].end());
  669:         auto ip = unique(v2v[v].begin(), v2v[v].end());
  670:         v2v[v].erase(ip, v2v[v].end());
  671:         //v2v[v].shrink_to_fit();       // automatically done when copied at return
  672:     }
  673:
  674:     return v2v;
  675: }
  676:
  677: vector<vector<int>> Mesh::Node2NodeGraph_1() const
  678: {
  679:     vector<vector<int>> v2v(_nnode, vector<int>(0));    // stores the vertex to vert
ex connections
  680:
  681:     for (int e = 0; e < _nelem; ++e)
  682:     {
  683:         int const basis = e * _nvert_e;                  // start of vertex connecti
vity of element e
  684:         for (int k = 0; k < _nvert_e; ++k)
  685:         {
  686:             int const v = _ia[basis + k];
  687:             for (int l = 0; l < _nvert_e; ++l)
  688:             {
  689:                 v2v[v].push_back(_ia[basis + l]);
  690:             }
  691:         }
  692:     }
  693:     // guarantee unique, ascending sorted entries per vertex
  694:     for (size_t v = 0; v < v2v.size(); ++v)
  695:     {
  696:         sort(v2v[v].begin(), v2v[v].end());
  697:         auto ip = unique(v2v[v].begin(), v2v[v].end());
  698:         v2v[v].erase(ip, v2v[v].end());
  699:         //v2v[v].shrink_to_fit();       // automatically done when copied at return
  700:     }
```

```
  701:
  702:     return v2v;
  703: }
  704:
  705:
  706: Mesh::Mesh(std::string const &fname)
  707:     : Mesh(2, 3, 3, 3) // two dimensions, 3 vertices, 3 dofs, 3 edges per element
  708: {
  709:     ReadVertexBasedMesh(fname);
  710:     DeriveEdgeFromVertexBased();        // Generate also the edge based information
  711:     //cout << " JJJJJJJJJ\n";
  712:     //DeriveEdgeFromVertexBased();
  713:     //cout << " KKKKKKKKKKK\n";
  714:     ////exit(-1);
  715: }
  716:
  717: void Mesh::ReadVertexBasedMesh(std::string const &fname)
  718: {
  719:     ifstream ifs(fname);
  720:     if (!(ifs.is_open() && ifs.good()))
  721:     {
  722:         cerr << "Mesh::ReadVertexBasedMesh: Error cannot open file " << fname << end
l;
  723:         assert(ifs.is_open());
  724:     }
  725:
  726:     int const OFFSET(1);             // Matlab to C indexing
  727:     cout << "ASCI file  " << fname << "  opened" << endl;
  728:
  729:     // Read some mesh constants
  730:     int nnode, ndim, nelem, nvert_e;
  731:     ifs >> nnode >> ndim >> nelem >> nvert_e;
  732:     cout << nnode << "  " << ndim << "  " << nelem << "  " << nvert_e << endl;
  733:     assert(ndim == 2 && nvert_e == 3);
  734:
  735:     // Allocate memory
  736:     Resize_Coords(nnode, ndim);                 // coordinates in 2D [nnode][ndim]
  737:     Resize_Connectivity(nelem, nvert_e);        // connectivity matrix [nelem][nvert
]
  738:
  739:     // Read coordinates
  740:     auto &xc = GetCoords();
  741:     for (int k = 0; k < nnode * ndim; ++k)
  742:     {
  743:         ifs >> xc[k];
  744:     }
  745:
  746:     // Read connectivity
  747:     auto &ia = GetConnectivity();
  748:     for (int k = 0; k < nelem * nvert_e; ++k)
  749:     {
  750:         ifs >> ia[k];
  751:         ia[k] -= OFFSET;                // Matlab to C indexing
  752:     }
  753:
  754:     // additional read of boundary information (only start/end point)
  755:     int nbedges;
  756:     ifs >> nbedges;
  757:
  758:     _bedges.resize(nbedges * 2);
  759:     for (int k = 0; k < nbedges * 2; ++k)
  760:     {
  761:         ifs >> _bedges[k];
  762:         _bedges[k] -= OFFSET;            // Matlab to C indexing
  763:     }
  764: // 2020-01-08 Check
  765:     int const DUMMY{-9876};
  766:     _sdedges.resize(nbedges * 2,DUMMY);
```

```
  767:     if (!ifs.eof())
  768:     {
  769:         cout << nbedges << endl;           // change to  while-loop
  770:         for (int k = 0; k < nbedges * 2; ++k)
  771:         {
  772:             ifs >> _sdedges.at(k);
  773:             _sdedges[k] -= OFFSET;            // Matlab to C indexing
  774:         }
  775:         //assert(ifs.eof());
  776:         assert(count(_sdedges.cbegin(),_sdedges.cend(),DUMMY)==0);
  777:     }
  778:     else
  779:     {
  780:         cout << "\n #######  no subdomain info of edges available!  #########\n";
  781:     }
  782:     cout << "\n End of File read " << _sdedges.at(2*nbedges-2) << " " << _sdedges.at
(2*nbedges-1)  << "\n" << endl;
  783:
  784:     return;
  785: }
  786:
  787: bool Mesh::Check_array_dimensions() const
  788: {
  789:     bool b_ia = static_cast<int>(_ia.size() / _nvert_e) == _nelem;
  790:     if (!b_ia)  cerr << "misfit: _nelem vs. _ia" << endl;
  791:
  792:     bool b_xc = static_cast<int>(_xc.size() / _ndim) == _nnode;
  793:     if (!b_xc)  cerr << "misfit: _nnode vs. _xc" << endl;
  794:
  795:     bool b_ea = static_cast<int>(_ea.size() / _nedge_e) == _nelem;
  796:     if (!b_ea)  cerr << "misfit: _nelem vs. _ea" << endl;
  797:
  798:     bool b_ed = static_cast<int>(_edges.size() / 2) == _nedge;
  799:     if (!b_ed)  cerr << "misfit: _nedge vs. _edges" << endl;
  800:
  801:
  802:     return b_ia && b_xc && b_ea && b_ed;
  803: }
  804:
  805: void Mesh::Del_EdgeConnectivity()
  806: {
  807:     _nedge = 0;               //!< number of edges in mesh
  808:     _edges.resize(0);         //!< edges of mesh (vertices ordered ascending)
  809:     _edges.shrink_to_fit();
  810:     _ea.resize(0);            //!< edge based element connectivity
  811:     _ea.shrink_to_fit();
  812:     _ebedges.resize(0);       //!< boundary edges [nbedges]
  813:     _ebedges.shrink_to_fit();
  814:     return;
  815: }
  816:
  817:
  818:
  819: // ###########################################################################
  820:
  821: RefinedMesh::RefinedMesh(Mesh const &cmesh, std::vector<bool> const &ibref)
  822: //: Mesh(cmesh), _cmesh(cmesh), _ibref(ibref), _nref(0), _vfathers(0)
  823:     : Mesh(cmesh), _ibref(ibref), _nref(0), _vfathers(0)
  824: {
  825:     if (_ibref.size() == 0)                  // refine all elements
  826:     {
  827:         //
  828:         RefineAllElements();
  829:     }
  830:     else
  831:     {
  832:         cout << endl << "  Adaptive Refinement not implemented yet." << endl;
  833:         assert(_ibref.size() != 0);
```

```
  834:     }
  835: }
  836:
  837: RefinedMesh::~RefinedMesh()
  838: {}
  839:
  840: Mesh RefinedMesh::RefineElements(std::vector<bool> const & /*ibref*/)
  841: {
  842:     Mesh new_mesh(_ndim, _nvert_e, _ndof_e, _nedge_e);
  843:     cout << " NOT IMPLEMENTED: Mesh::RefineElements" << endl;
  844:
  845: ////  initialize new coorsinates with the old one
  846:     //auto new_coords = new_mesh.GetCoords();
  847:     //new_coords = _xc;                       // copy coordinates from old mesh
  848:
  849: //// access vertex connectivite, edge connectiviy and edge information of new mesh
  850:     //auto new_ia = new_mesh.GetConnectivity();
  851:     //auto new_ea = new_mesh.GetEdgeConnectivity();
  852:     //auto new_edges = new_mesh.GetEdges();
  853:
  854: ////  storing the parents of edges and vertices
  855:
  856:
  857:     //assert( new_ia.size()== new_ea.size() );
  858:     //new_mesh.SetNnode( new_coords.size() );
  859:     //new_mesh.SetNelem( new_ia.size()/3 );
  860:     //new_mesh._nedge = new_edges.size()/2;
  861:
  862:     return new_mesh;
  863: }
  864:
  865: //JF
  866: void RefinedMesh::RefineAllElements(int nref)
  867: {
  868:     cout << "\n#############   Refine Mesh " << nref << " times ";
  869:     double tstart = clock();
  870:     DeriveEdgeFromVertexBased();          // ensure that edge information is availab
le
  871:
  872:     for (int kr = 0; kr < nref; ++kr)
  873:     {
  874:         //DeriveEdgeFromVertexBased();          // ensure that edge information is a
vailable // GH: not needed in each loop
  875:
  876:         auto old_ea(_ea);                     // save old edge connectivity
  877:         auto old_edges(_edges);               // save old edges
  878:         auto old_nedges(Nedges());
  879:         auto old_nnodes(Nnodes());
  880:         auto old_nelems(Nelems());
  881:
  882:         //  the new vertices will be appended to the coordinates in _xc
  883:
  884:         vector<int> edge_sons(2 * old_nedges); // 2 sons for each edge
  885:
  886:         //   --  Derive the fine edges ---
  887:         int new_nedge = 2 * old_nedges + 3 * old_nelems; // #edges in new mesh
  888:         int new_nelem = 4 * old_nelems;     // #elements in new mesh
  889:         int new_nnode = old_nnodes + old_nedges; // #nodes in new mesh
  890:
  891:         _xc.reserve(2 * new_nnode);
  892:         // store the 2 fathers of each vertex (equal fathers denote original coarse
vertex)
  893:         _vfathers.resize(2 * old_nnodes);
  894:         for (int vc = 0; vc < old_nnodes; ++vc)
  895:         {
  896:             _vfathers[2 * vc  ] = vc;    // equal fathers denote original coarse ver
tex
  897:             _vfathers[2 * vc + 1] = vc;
```

```
  898:         }
  899:
  900:         _ia.clear();
  901:         _ea.clear();
  902:         _ea.resize(new_nelem * 3);
  903:         _edges.clear();
  904:         _edges.resize(2 * new_nedge);          // vertices of edges [v_0, v_1;v_0, v
_1; ...]
  905:         vector<int> e_son(2 * old_nedges); // sons of coarse edges [s_0, s_1; s_0, s
_1; ...]
  906:
  907:         // split all coarse edges and append the new nodes
  908:         int kf = 0;                                // index of edges in fine mesh
  909:         int vf = old_nnodes;                // index of new vertex in fine grid
  910:         for (int kc = 0; kc < old_nedges; ++kc)    // index of edges in coarse mesh
  911:         {
  912:             //
  913:             int v1 = old_edges[2 * kc];         // vertices of old edge
  914:             int v2 = old_edges[2 * kc + 1];
  915:             // append coordinates of new vertex
  916:             double xf = 0.5 * ( _xc[2 * v1  ] + _xc[2 * v2  ] );
  917:             double yf = 0.5 * ( _xc[2 * v1 + 1] + _xc[2 * v2 + 1] );
  918:             _xc.push_back(xf);
  919:             _xc.push_back(yf);
  920:             // fathers of vertex  vf
  921:             _vfathers.push_back(v1);
  922:             _vfathers.push_back(v2);
  923:
  924:             // split old edge into two edges
  925:             _edges[2 * kf    ] = v1;             // coarse vertex 1
  926:             _edges[2 * kf + 1] = vf;             //   to new fine vertex
  927:             e_son[2 * kc    ]  = kf;             // son edge
  928:             ++kf;
  929:             _edges[2 * kf    ] = vf;             // new fine vertex
  930:             _edges[2 * kf + 1] = v2;             //   to coarse vertex 2
  931:             e_son[2 * kc + 1]  = kf;             // son edge
  932:
  933:             ++vf;
  934:             ++kf;
  935:         }
  936:         _xc.shrink_to_fit();
  937:         _vfathers.shrink_to_fit();
  938:
  939:         // -- derive the fine mesh elements --
  940:         //    creates additional fine edges
  941:
  942:         for (int kc = 0; kc < old_nelems; ++kc)    // index of elements in coarse mes
h
  943:         {
  944:             array<array<int, 3>, 3 * 2> boundary; // fine scale vertices and edges a
s boundary of old element
  945:             //boundary[ ][0], boundary[ ][1] ..vertices boundary[ ][2] edge
  946:
  947:             for (int j = 0; j < 3; ++j)            // each edge in element
  948:             {
  949:                 int ce = old_ea[3 * kc + j];       // coarse edge number
  950:
  951:                 int s1 = e_son[2 * ce    ];        // son edges of that coarse edge
  952:                 int s2 = e_son[2 * ce + 1];
  953:                 boundary[2 * j][2] = s1;           //add boundary edge
  954:                 boundary[2 * j][0] = _edges[2 * s1 + 0];
  955:                 boundary[2 * j][1] = _edges[2 * s1 + 1];
  956:                 if (boundary[2 * j][0] > boundary[2 * j][1]) swap(boundary[2 * j][0]
, boundary[2 * j][1]);          // fine vertices always in 2nd entry
  957:                 boundary[2 * j + 1][2] = s2;       //add boundary edge
  958:                 boundary[2 * j + 1][0] = _edges[2 * s2 + 0];
  959:                 boundary[2 * j + 1][1] = _edges[2 * s2 + 1];
  960:                 if (boundary[2 * j + 1][0] > boundary[2 * j + 1][1]) swap(boundary[2
```

```
 * j + 1][0], boundary[2 * j + 1][1]);
  961:             }
  962:
  963:             sort(boundary.begin(), boundary.end());             // sort -> edges wit
h same coarse vertex will be neighbors
  964:
  965:             int interior_1 = 2 * old_nedges + kc * 3;   // add interior edges
  966:             int interior_2 = 2 * old_nedges + kc * 3 + 1;
  967:             int interior_3 = 2 * old_nedges + kc * 3 + 2;
  968:
  969:             _edges[interior_1 * 2    ] = boundary[0][1]; // add interior edges
  970:             _edges[interior_1 * 2 + 1] = boundary[1][1];
  971:
  972:             _edges[interior_2 * 2    ] = boundary[2][1];
  973:             _edges[interior_2 * 2 + 1] = boundary[3][1];
  974:
  975:             _edges[interior_3 * 2    ] = boundary[4][1];
  976:             _edges[interior_3 * 2 + 1] = boundary[5][1];
  977:
  978:             _ea[kc * 3 * 4    ] = boundary[0][2];       // add 4 new elements with 3
 edges for every old element
  979:             _ea[kc * 3 * 4 + 1] = boundary[1][2];
  980:             _ea[kc * 3 * 4 + 2] = interior_1;
  981:
  982:             _ea[kc * 3 * 4 + 3] = boundary[2][2];
  983:             _ea[kc * 3 * 4 + 4] = boundary[3][2];
  984:             _ea[kc * 3 * 4 + 5] = interior_2;
  985:
  986:             _ea[kc * 3 * 4 + 6] = boundary[4][2];
  987:             _ea[kc * 3 * 4 + 7] = boundary[5][2];
  988:             _ea[kc * 3 * 4 + 8] = interior_3;
  989:
  990:             _ea[kc * 3 * 4 + 9] = interior_1;
  991:             _ea[kc * 3 * 4 + 10] = interior_2;
  992:             _ea[kc * 3 * 4 + 11] = interior_3;
  993:         }
  994:
  995: // GH: ToDo:  _bedges  has to updated for the new mesh //!< boundary edges [nbedges]
[2] storing start/end vertex
  996: //     Pass the refinement information to the boundary edges (edge based)
  997:          auto old_ebedges(_ebedges);            // save original boundary edges [nbedg
es] (edge based storage)
  998:         unsigned int old_nbedges(old_ebedges.size());
  999:
 1000:         _ebedges.resize(2 * old_nbedges);    // each old boundary edge will be bisec
ted
 1001:         unsigned int kn = 0;                  // index of new boundary edges
 1002:         for (unsigned int ke = 0; ke < old_nbedges; ++ke)   // index of old boundary
 edges
 1003:         {
 1004:             const auto kc = old_ebedges[ke];
 1005:             _ebedges[kn] = e_son[2 * kc    ];
 1006:             ++kn;
 1007:             _ebedges[kn] = e_son[2 * kc + 1];
 1008:             ++kn;
 1009:         }
 1010: // HG
 1011:         // set new mesh parameters
 1012:         SetNelem(new_nelem);
 1013:         SetNnode(new_nnode);
 1014:         SetNedge(new_nedge);
 1015:
 1016: #ifdef CUTHILL_MCKEE
 1017:         {
 1018: // Cuthill-McKee reordering
 1019: //     Increases mesh generation time by factor 5 -  but solver is faster.
 1020:             auto const perm = cuthill_mckee_reordering(_edges);
 1021:             PermuteVertices_EdgeBased(perm);
```

```
1022:         }
1023: #endif
1024:
1025:         DeriveVertexFromEdgeBased();
1026:         assert( RefinedMesh::Check_array_dimensions() );
1027:
1028:         ++_nref;                              // track the number of refinements
1029:     }
1030:
1031:     double duration = (clock() - tstart) / CLOCKS_PER_SEC;
1032:     cout << "finished in  " <<  duration  << " sec.    ########\n";
1033:
1034:     return;
1035: }
1036:
1037:
1038: void Mesh::PermuteVertices_EdgeBased(vector<int> const &old2new)
1039: {
1040: //      permute vertices _edges
1041:     auto const edges_old(_edges);
1042:     for (size_t k = 0; k < _edges.size(); k += 2)
1043:     {
1044:         _edges[k    ] = old2new[edges_old[k    ]];
1045:         _edges[k + 1] = old2new[edges_old[k + 1]];
1046:         if (_edges[k] > _edges[k + 1])
1047:             swap(_edges[k], _edges[k + 1]);
1048:     }
1049: //      permute coordinates
1050:     auto const coord_old(_xc);
1051:     for (size_t k = 0; k < _xc.size() / 2; ++k)
1052:     {
1053:         _xc[2 * old2new[k]    ] = coord_old[2 * k    ];
1054:         _xc[2 * old2new[k] + 1] = coord_old[2 * k + 1];
1055:     }
1056:     return;
1057: }
1058:
1059:
1060: void RefinedMesh::PermuteVertices_EdgeBased(vector<int> const &old2new)
1061: {
1062:     Mesh::PermuteVertices_EdgeBased(old2new);
1063: //      permute fathers of a vertex
1064:     auto const old_fathers(_vfathers);
1065:     for (size_t k = 0; k < _vfathers.size() / 2; ++k)
1066:     {
1067:         _vfathers[2 * old2new[k]    ] = old_fathers[2 * k    ];
1068:         _vfathers[2 * old2new[k] + 1] = old_fathers[2 * k + 1];
1069:     }
1070:     return;
1071: }
1072:
1073:
1074: bool RefinedMesh::Check_array_dimensions() const
1075: {
1076:     const bool bp = Mesh::Check_array_dimensions();
1077:
1078:     const bool bvf = (static_cast<int>(_vfathers.size()) / 2 == Nnodes());
1079:
1080:     return bp && bvf;
1081:
1082: }
1083: // ########################################################################
1084:
1085:
1086: gMesh_Hierarchy::gMesh_Hierarchy(Mesh const &cmesh, int const nlevel)
1087:     : _gmesh(max(1, nlevel))
1088: {
1089:     _gmesh[0] = make_shared<Mesh>(cmesh);
```

```
1090:     for (int lev = 1; lev < nlevel; ++lev)
1091:     {
1092:         _gmesh.at(lev) = make_shared<RefinedMesh>( *_gmesh.at(lev - 1) );
1093:         //auto vv=_gmesh[lev]->GetFathersOfVertices();
1094:         //cout << " :: "<< vv.size() <<endl;
1095:     }
1096:     for (size_t lev = 0; lev < _gmesh.size(); ++lev)
1097:     {
1098:         _gmesh[lev]->Del_EdgeConnectivity();
1099:     }
1100: }
1101:
1102:
1103: // ########################################################################
1104: Mesh_2d_3_square::Mesh_2d_3_square(int nx, int ny, int myid, int procx, int procy)
1105:     : Mesh(2, 3, 3, 3), // two dimensions, 3 vertices, 3 dofs, 3 edges per element
1106:       _myid(myid), _procx(procx), _procy(procy), _neigh{{ -1, -1, -1, -1}}, _color(0
),
1107: _xl(0.0), _xr(1.0), _yb(0.0), _yt(1.0), _nx(nx), _ny(ny)
1108: {
1109:     //void IniGeom(int const myid, int const procx, int const procy, int neigh[], in
t &color)
1110:     int const ky = _myid / _procx;
1111:     int const kx = _myid % _procy;      //    MOD(myid,procx)
1112:     // Determine the neighbors of domain/rank myid
1113:     _neigh[0] = (ky == 0)       ?  -1 : _myid - _procx;    //   South
1114:     _neigh[1] = (kx == _procx - 1) ?  -1 : _myid + 1;      //   East
1115:     _neigh[2] = (ky == _procy - 1) ?  -1 : _myid + _procx; //   North
1116:     _neigh[3] = (kx == 0)       ?  -1 : _myid - 1;         //   West
1117:
1118:     _color = (kx + ky) & 1 ;
1119:
1120:     // subdomain is part of unit square
1121:     double const hx = 1. / _procx;
1122:     double const hy = 1. / _procy;
1123:     _xl = kx * hx;                      //  left
1124:     _xr = (kx + 1) * hx;                //  right
1125:     _yb = ky * hy;                      //  bottom
1126:     _yt = (ky + 1) * hy;                //  top
1127:
1128:     // Calculate coordinates
1129:     int const nnode = (_nx + 1) * (_ny + 1); // number of nodes
1130:     Resize_Coords(nnode, 2);                 // coordinates in 2D [nnode][ndim]
1131:     GetCoordsInRectangle(_nx, _ny, _xl, _xr, _yb, _yt, GetCoords().data());
1132:
1133:     // Calculate element connectivity (linear triangles)
1134:     int const nelem = 2 * _nx * _ny;         // number of elements
1135:     Resize_Connectivity(nelem, 3);           // connectivity matrix [nelem][3]
1136:     GetConnectivityInRectangle(_nx, _ny, GetConnectivity().data());
1137:
1138:     return;
1139: }
1140:
1141: Mesh_2d_3_square::~Mesh_2d_3_square()
1142: {}
1143:
1144:
1145: void Mesh_2d_3_square::SetU(std::vector<double> &u) const
1146: {
1147:     int dx    = _nx + 1;
1148:     for (int j = 0; j <= _ny; ++j)
1149:     {
1150:         int k = j * dx;
1151:         for (int i = 0; i <= _nx; ++i, ++k)
1152:         {
1153:             u[k] = 0.0;
1154:         }
1155:     }
```

```
1156:
1157: }
1158:
1159: void Mesh_2d_3_square::SetF(std::vector<double> &f) const
1160: {
1161:     int dx    = _nx + 1;
1162:     for (int j = 0; j <= _ny; ++j)
1163:     {
1164:         int k = j * dx;
1165:         for (int i = 0; i <= _nx; ++i, ++k)
1166:         {
1167:             f[k] = 1.0;
1168:         }
1169:     }
1170:
1171: }
1172:
1173:
1174: std::vector<int> Mesh_2d_3_square::Index_DirichletNodes() const
1175: {
1176:     int const dx = 1,
1177:               dy = _nx + 1,
1178:               bl  = 0,
1179:               br  = _nx,
1180:               tl  = _ny * (_nx + 1),
1181:               tr  = (_ny + 1) * (_nx + 1) - 1;
1182:     int const start[4] = { bl, br, tl, bl};
1183:     int const   end[4] = { br, tr, tr, tl};
1184:     int const  step[4] = { dx, dy, dx, dy};
1185:
1186:     vector<int> idx(0);
1187:     for (int j = 0; j < 4; j++)
1188:     {
1189:         if (_neigh[j] < 0)
1190:         {
1191:             for (int i = start[j]; i <= end[j]; i += step[j])
1192:             {
1193:                 idx.push_back(i);        // node i is Dirichlet node
1194:             }
1195:         }
1196:     }
1197:     //    remove multiple elements
1198:     sort(idx.begin(), idx.end());                              // sort
1199:     idx.erase( unique(idx.begin(), idx.end()), idx.end() ); // remove duplicate data
1200:
1201:     return idx;
1202: }
1203:
1204: vector<int> Mesh_2d_3_square::Index_BoundaryNodes() const
1205: {
1206:     cerr << "Warning: Funktion not implemented.  " << __FILE__ << "." << __LINE__ <<
 endl;
1207:     return Index_BoundaryNodes();
1208: }
1209:
1210:
1211:
1212:
1213: void Mesh_2d_3_square::SaveVectorP(std::string const &name, vector<double> const &u)
 const
1214: {
1215: //  construct the file name for subdomain myid
1216:     const string tmp( std::to_string(_myid / 100) + to_string((_myid % 100) / 10) +
to_string(_myid % 10) );
1217:
1218:     const string namep = name + "." + tmp;
1219:     ofstream ff(namep.c_str());
1220:     ff.precision(6);
```

```
 1221:     ff.setf(ios::fixed, ios::floatfield);
 1222:
 1223:     // assumes tensor product grid in unit square; rowise numbered (as generated in
class constructor)
 1224:     // output is provided for tensor product grid visualization ( similar to Matlab-
surf() )
 1225:     auto const &xc = GetCoords();
 1226:     int k = 0;
 1227:     for (int j = 0; j <= _ny; ++j)
 1228:     {
 1229:         for (int i = 0; i <= _nx; ++i, ++k)
 1230:             ff << xc[2 * k + 0] << "   " << xc[2 * k + 1] << "   " << u[k] << endl;
 1231:         ff << endl;
 1232:     }
 1233:
 1234:     ff.close();
 1235:     return;
 1236: }
 1237:
 1238: void Mesh_2d_3_square::GetCoordsInRectangle(int const nx, int const ny,
 1239:         double const xl, double const xr, double const yb, double const yt,
 1240:         double xc[])
 1241: {
 1242:     const double hx = (xr - xl) / nx,
 1243:                  hy = (yt - yb) / ny;
 1244:
 1245:     int k = 0;
 1246:     for (int j = 0; j <= ny; ++j)
 1247:     {
 1248:         const double y0 = yb + j * hy;
 1249:         for (int i = 0; i <= nx; ++i, k += 2)
 1250:         {
 1251:             xc[k  ] = xl + i * hx;
 1252:             xc[k + 1] = y0;
 1253:         }
 1254:     }
 1255:
 1256:     return;
 1257: }
 1258:
 1259: void Mesh_2d_3_square::GetConnectivityInRectangle(int const nx, int const ny, int ia
[])
 1260: {
 1261:     const int dx = nx + 1;
 1262:     int k  = 0;
 1263:     int l  = 0;
 1264:     for (int j = 0; j < ny; ++j, ++k)
 1265:     {
 1266:         for (int i = 0; i < nx; ++i, ++k)
 1267:         {
 1268:             ia[l  ] = k;
 1269:             ia[l + 1] = k + 1;
 1270:             ia[l + 2] = k + dx + 1;
 1271:             l += 3;
 1272:             ia[l  ] = k;
 1273:             ia[l + 1] = k + dx;
 1274:             ia[l + 2] = k + dx + 1;
 1275:             l += 3;
 1276:         }
 1277:     }
 1278:     return;
 1279: }
 1280:
 1281: // ########################################################################
 1282:
```

```
    1: #ifndef GEOM_FILE
    2: #define GEOM_FILE
    3: #include <array>
    4: #include <functional>              // function; C++11
    5: #include <iostream>
    6: #include <memory>                   // shared_ptr
    7: #include <string>
    8: #include <vector>
    9:
   10: /**
   11:  * Basis class for finite element meshes.
   12:  */
   13: class Mesh
   14: {
   15: public:
   16:     /**
   17:      * Constructor initializing the members with default values.
   18:      *
   19:      * @param[in] ndim  space dimensions (dimension for coordinates)
   20:      * @param[in] nvert_e  number of vertices per element (dimension for connectivi
ty)
   21:      * @param[in] ndof_e   degrees of freedom per element (= @p nvert_e for linear
elements)
   22:      * @param[in] nedge_e  number of edges per element (= @p nvert_e for linear ele
ments in 2D)
   23:      */
   24:     explicit Mesh(int ndim, int nvert_e = 0, int ndof_e = 0, int nedge_e = 0);
   25:
   26:     __attribute__((noinline))
   27:     Mesh(Mesh const &) = default;
   28:
   29:     Mesh &operator=(Mesh const &) = delete;
   30:
   31:
   32:     /**
   33:      * Destructor.
   34:      *
   35:      * See clang warning on
   36:      * <a href="https://stackoverflow.com/questions/28786473/clang-no-out-of-line-vi
rtual-method-definitions-pure-abstract-c-class/40550578">weak-vtables</a>.
   37:      */
   38:     virtual ~Mesh();
   39:
   40:     /**
   41:      * Reads mesh data from a binary file.
   42:      *
   43:      * File format, see ascii_write_mesh.m
   44:      *
   45:      * @param[in] fname file name
   46:     */
   47:     explicit Mesh(std::string const &fname);
   48:
   49:     /**
   50:      * Reads mesh data from a binary file.
   51:      *
   52:      * File format, see ascii_write_mesh.m
   53:      *
   54:      * @param[in] fname file name
   55:     */
   56:     void ReadVertexBasedMesh(std::string const &fname);
   57:
   58:     /**
   59:      * Number of finite elements in (sub)domain.
   60:      * @return number of elements.
   61:      */
   62:     int Nelems() const
   63:     {
   64:         return _nelem;
```

```
    65:     }
    66:
    67:     /**
    68:      * Global number of vertices for each finite element.
    69:      * @return number of vertices per element.
    70:      */
    71:     int NverticesElements() const
    72:     {
    73:         return _nvert_e;
    74:     }
    75:
    76:     /**
    77:      * Global number of degrees of freedom (dof) for each finite element.
    78:      * @return degrees of freedom per element.
    79:      */
    80:     int NdofsElement() const
    81:     {
    82:         return _ndof_e;
    83:     }
    84:
    85:     /**
    86:      * Number of vertices in mesh.
    87:      * @return number of vertices.
    88:      */
    89:     int Nnodes() const
    90:     {
    91:         return _nnode;
    92:     }
    93:
    94:     /**
    95:      * Space dimension.
    96:      * @return number of dimensions.
    97:      */
    98:     int Ndims() const
    99:     {
   100:         return _ndim;
   101:     }
   102:
   103:     /**
   104:      * (Re-)Allocates memory for the element connectivity and redefines the appropri
ate dimensions.
   105:      *
   106:      * @param[in] nelem    number of elements
   107:      * @param[in] nvert_e  number of vertices per element
   108:      */
   109:     void Resize_Connectivity(int nelem, int nvert_e)
   110:     {
   111:         SetNelem(nelem);                // number of elements
   112:         SetNverticesElement(nvert_e);  // vertices per element
   113:         _ia.resize(nelem * nvert_e);
   114:     }
   115:
   116:     /**
   117:      * Read connectivity information (g1,g2,g3)_i.
   118:      * @return connectivity vector [nelems*ndofs].
   119:      */
   120:     const std::vector<int>  &GetConnectivity() const
   121:     {
   122:         return _ia;
   123:     }
   124:
   125:     /**
   126:      * Access/Change connectivity information (g1,g2,g3)_i.
   127:      * @return connectivity vector [nelems*ndofs].
   128:      */
   129:     std::vector<int>  &GetConnectivity()
   130:     {
   131:         return _ia;
```

```
  132:     }
  133:
  134:     /**
  135:      * (Re-)Allocates memory for the element connectivity and redefines the appropri
ate dimensions.
  136:      *
  137:      * @param[in] nnodes    number of nodes
  138:      * @param[in] ndim      space dimension
  139:      */
  140:     void Resize_Coords(int nnodes, int ndim)
  141:     {
  142:         SetNnode(nnodes);       // number of nodes
  143:         SetNdim(ndim);          // space dimension
  144:         _xc.resize(nnodes * ndim);
  145:     }
  146:
  147:     /**
  148:      * Read coordinates of vertices (x,y)_i.
  149:      * @return coordinates vector [nnodes*2].
  150:      */
  151:     const std::vector<double> &GetCoords() const
  152:     {
  153:         return _xc;
  154:     }
  155:
  156:     /**
  157:      * Access/Change coordinates of vertices (x,y)_i.
  158:      * @return coordinates vector [nnodes*2].
  159:      */
  160:     std::vector<double> &GetCoords()
  161:     {
  162:         return _xc;
  163:     }
  164:
  165:     /**
  166:      * Calculate values in vector @p v via function @p func(x,y)
  167:      * @param[in] v     vector
  168:      * @param[in] func  function of (x,y) returning a double value.
  169:      */
  170:     void SetValues(std::vector<double> &v, const std::function<double(double, double
)> &func) const;
  171:     void SetBoundaryValues(std::vector<double> &v, const std::function<double(double
, double)> &func) const;
  172:     void SetDirchletValues(std::vector<double> &v, const std::function<double(double
, double)> &func) const;
  173:
  174:     /**
  175:      * Prints the information for a finite element mesh
  176:      */
  177:     void Debug() const;
  178:
  179:     /**
  180:      * Prints the edge based information for a finite element mesh
  181:      */
  182:     void DebugEdgeBased() const;
  183:
  184:     /**
  185:      * Determines the indices of those vertices with Dirichlet boundary conditions
  186:      * @return index vector.
  187:      */
  188:     virtual std::vector<int> Index_DirichletNodes() const;
  189:     virtual std::vector<int> Index_BoundaryNodes() const;
  190:
  191:     /**
  192:      * Write vector @p v together with its mesh information to an ASCii file @p fnam
e.
  193:      *
  194:      * The data are written in C-style.
```

```
  195:       *
  196:       * @param[in] fname  file name
  197:       * @param[in] v      vector
  198:       */
  199:      void Write_ascii_matlab(std::string const &fname, std::vector<double> const &v)
const;
  200:
  201:      /**
  202:       * Exports the mesh information to ASCii files  @p basename + {_coords|_elements
}.txt.
  203:       *
  204:       * The data are written in C-style.
  205:       *
  206:       * @param[in] basename  first part of file names
  207:       */
  208:      void Export_scicomp(std::string const &basename) const;
  209:
  210:      /**
  211:       * Visualize @p v together with its mesh information via matlab or octave.
  212:       *
  213:       * Comment/uncomment those code lines in method Mesh:Visualize (geom.cpp)
  214:       * that are supported on your system.
  215:       *
  216:       * @param[in] v      vector
  217:       *
  218:       * @warning matlab files ascii_read_meshvector.m  visualize_results.m
  219:       *          must be in the executing directory.
  220:       */
  221:      void Visualize(std::vector<double> const &v) const;
  222:
  223:      /**
  224:       * Global number of edges.
  225:       * @return number of edges in mesh.
  226:       */
  227:      int Nedges() const
  228:      {
  229:          return _nedge;
  230:      }
  231:
  232:      /**
  233:       * Global number of edges for each finite element.
  234:       * @return number of edges per element.
  235:       */
  236:      int NedgesElements() const
  237:      {
  238:          return _nedge_e;
  239:      }
  240:
  241:      /**
  242:       * Read edge connectivity information (e1,e2,e3)_i.
  243:       * @return edge connectivity vector [nelems*_nedge_e].
  244:       */
  245:      const std::vector<int>  &GetEdgeConnectivity() const
  246:      {
  247:          return _ea;
  248:      }
  249:
  250:      /**
  251:       * Access/Change edge connectivity information (e1,e2,e3)_i.
  252:       * @return edge connectivity vector [nelems*_nedge_e].
  253:       */
  254:      std::vector<int>  &GetEdgeConnectivity()
  255:      {
  256:          return _ea;
  257:      }
  258:
  259:      /**
  260:       * Read edge information (v1,v2)_i.
```

```
261:      * @return edge connectivity vector [_nedge*2].
262:      */
263:     const std::vector<int>  &GetEdges() const
264:     {
265:         return _edges;
266:     }
267:
268:     /**
269:      * Access/Change edge information (v1,v2)_i.
270:      * @return edge connectivity vector [_nedge*2].
271:      */
272:     std::vector<int>  &GetEdges()
273:     {
274:         return _edges;
275:     }
276:
277:     /**
278:      * Determines all node to node connections from the vertex based mesh.
279:       *
280:      * @return vector[k][] containing all connections of vertex k, including to itse
lf.
281:      */
282:     std::vector<std::vector<int>> Node2NodeGraph() const
283:     {
284:         //// Check version 2 wrt. version 1
285:         //auto v1=Node2NodeGraph_1();
286:         //auto v2=Node2NodeGraph_2();
287:         //if ( equal(v1.cbegin(),v1.cend(),v2.begin()) )
288:         //{
289:         //std::cout << "\nidentical Versions\n";
290:         //}
291:         //else
292:         //{
293:         //std::cout << "\nE R R O R   in Versions\n";
294:         //}
295:
296:         //return Node2NodeGraph_1();
297:         return Node2NodeGraph_2();        // 2 times faster than version 1
298:     }
299:
300:     /**
301:      * Accesses the father-of-nodes relation.
302:       *
303:      * @return  vector of length 0 because no relation available.
304:       *
305:      */
306:     virtual std::vector<int> const &GetFathersOfVertices() const
307:     {
308:         return _dummy;
309:     }
310:
311:     /**
312:      * Deletes all edge connectivity information (saves memory).
313:      */
314:     void Del_EdgeConnectivity();
315:
316: protected:
317:     //public:
318:     void SetNelem(int nelem)
319:     {
320:         _nelem = nelem;
321:     }
322:
323:     void SetNverticesElement(int nvert)
324:     {
325:         _nvert_e = nvert;
326:     }
327:
```

```
  328:     void SetNdofsElement(int ndof)
  329:     {
  330:         _ndof_e = ndof;
  331:     }
  332:
  333:     void SetNnode(int nnode)
  334:     {
  335:         _nnode = nnode;
  336:     }
  337:
  338:     void SetNdim(int ndim)
  339:     {
  340:         _ndim = ndim;
  341:     }
  342:
  343:     void SetNedge(int nedge)
  344:     {
  345:         _nedge = nedge;
  346:     }
  347:
  348:     /**
  349:      * Reads vertex based mesh data from a binary file.
  350:      *
  351:      * File format, see ascii_write_mesh.m
  352:      *
  353:      * @param[in] fname file name
  354:      */
  355:     void ReadVectexBasedMesh(std::string const &fname);
  356:
  357:     /**
  358:      * The vertex based mesh data are used to derive the edge based data.
  359:      *
  360:      *  @warning Exactly 3 vertices, 3 edges per element are assumed (linear triangl
e in 2D)
  361:      */
  362:     void DeriveEdgeFromVertexBased()
  363:     {
  364:         //DeriveEdgeFromVertexBased_slow();
  365:         //DeriveEdgeFromVertexBased_fast();
  366:         DeriveEdgeFromVertexBased_fast_2();
  367:     }
  368:     void DeriveEdgeFromVertexBased_slow();
  369:     void DeriveEdgeFromVertexBased_fast();
  370:     void DeriveEdgeFromVertexBased_fast_2();
  371:
  372:
  373:
  374:     /**
  375:      * The edge based mesh data are used to derive the vertex based data.
  376:       *
  377:       *  @warning Exactly 3 vertices, 3 edges per element are assumed (linear triang
le in 2D)
  378:      */
  379:     void DeriveVertexFromEdgeBased();
  380:
  381:     /**
  382:      * Determines the indices of those vertices with Dirichlet boundary conditions
  383:      * @return index vector.
  384:      */
  385:     int Nnbedges() const
  386:     {
  387:         return static_cast<int>(_bedges.size());
  388:     }
  389:
  390:     /**
  391:      * Checks whether the array dimensions fit to their appropriate size parameters
  392:      * @return index vector.
  393:      */
```

```
  394:     virtual bool Check_array_dimensions() const;
  395:
  396:     /**
  397:      * Permutes the vertex information in an edge based mesh.
  398:      *
  399:      * @param[in] old2new   new indices of original vertices.
  400:      */
  401:     void PermuteVertices_EdgeBased(std::vector<int> const &old2new);
  402:
  403: private:
  404:     /**
  405:      * Determines all node to node connections from the vertex based mesh.
  406:      *
  407:      * @return vector[k][] containing all connections of vertex k, including to itse
lf.
  408:      */
  409:     std::vector<std::vector<int>> Node2NodeGraph_1() const;  // is correct
  410:
  411:     /**
  412:      * Determines all node to node connections from the vertex based mesh.
  413:      *
  414:      * Faster than @p Node2NodeGraph_1().
  415:      *
  416:      * @return vector[k][] containing all connections of vertex k, including to itse
lf.
  417:      */
  418:     std::vector<std::vector<int>> Node2NodeGraph_2() const;  // is correct
  419:
  420:     //private:
  421: protected:
  422:     int _nelem;         //!< number elements
  423:     int _nvert_e;       //!< number of vertices per element
  424:     int _ndof_e;        //!< degrees of freedom (d.o.f.) per element
  425:     int _nnode;         //!< number nodes/vertices
  426:     int _ndim;          //!< space dimension of the problem (1, 2, or 3)
  427:     std::vector<int> _ia;    //!< element connectivity
  428:     std::vector<double> _xc; //!< coordinates
  429:
  430: protected:
  431:     // B.C.
  432:     std::vector<int> _bedges;     //!< boundary edges [nbedges][2] storing start/end
 vertex
  433: // 2020-01-08
  434:     std::vector<int> _sdedges;    //!< boundary edges [nbedges][2] with left/right s
ubdomain number
  435:
  436:     //private:
  437: protected:
  438:     // edge based connectivity
  439:     int _nedge;             //!< number of edges in mesh
  440:     int _nedge_e;           //!< number of edges per element
  441:     std::vector<int> _edges; //!< edges of mesh (vertices ordered ascending)
  442:     std::vector<int> _ea;    //!< edge based element connectivity
  443:     // B.C.
  444:     std::vector<int> _ebedges; //!< boundary edges [nbedges]
  445:
  446: private:
  447:     const std::vector<int> _dummy; //!< empty dummy vector
  448:
  449: };
  450:
  451:
  452: // ********************************************************************
  453:
  454: class RefinedMesh: public Mesh
  455: {
  456: public:
  457:     /**
```

```
458:      * Constructs a refined mesh according to the marked elements in @p ibref.
459:      *
460:      * If the vector @p ibref has size 0 then all elements will be refined.
461:      *
462:      * @param[in] cmesh  original mesh for coarsening.
463:      * @param[in] ibref  vector containing True/False regarding refinement for each
element
464:      *
465:      */
466:     //explicit RefinedMesh(Mesh const &cmesh, std::vector<bool> const &ibref = std::
vector<bool>(0));
467:     RefinedMesh(Mesh const &cmesh, std::vector<bool> const &ibref);
468:     //RefinedMesh(Mesh const &cmesh, std::vector<bool> const &ibref);
469:
470:     /**
471:     * Constructs a refined mesh by regulare refinement of all elements.
472:     *
473:     * @param[in] cmesh  original mesh for coarsening.
474:     *
475:     */
476:     explicit RefinedMesh(Mesh const &cmesh)
477:         : RefinedMesh(cmesh, std::vector<bool>(0))
478:     {}
479:
480:
481:     RefinedMesh(RefinedMesh const &) = delete;
482:     //RefinedMesh(RefinedMesh const&&) = delete;
483:
484:     RefinedMesh &operator=(RefinedMesh const &) = delete;
485:     //RefinedMesh& operator=(RefinedMesh const&&) = delete;
486:
487:     /**
488:      * Destructor.
489:      */
490:     virtual ~RefinedMesh() override;
491:
492:     /**
493:      * Refines the mesh according to the marked elements.
494:      *
495:      * @param[in] ibref  vector containing True/False regarding refinement for each
element
496:      *
497:      * @return the refined mesh
498:      *
499:      */
500:     Mesh RefineElements(std::vector<bool> const &ibref);
501:
502:     /**
503:      * Refines all elements in the actual mesh.
504:      *
505:      * @param[in] nref  number of regular refinements to perform
506:      *
507:      */
508:     void RefineAllElements(int nref = 1);
509:
510:     /**
511:      * Accesses the father-of-nodes relation.
512:       *
513:      * @return  father-of-nodes relation [nnodes][2]
514:       *
515:      */
516:     std::vector<int> const &GetFathersOfVertices() const override
517:     {
518:         return _vfathers;
519:     }
520:
521: protected:
522:     /**
```

```
523:      * Checks whether the array dimensions fit to their appropriate size parameters
524:      * @return index vector.
525:      */
526:     bool Check_array_dimensions() const override;
527:
528:     /**
529:      * Permutes the vertex information in an edge based mesh.
530:      *
531:      * @param[in] old2new   new indices of original vertices.
532:      */
533:     void PermuteVertices_EdgeBased(std::vector<int> const &old2new);
534:
535:
536: private:
537:     //Mesh const               & _cmesh; //!< coarse mesh
538:     std::vector<bool> const   _ibref; //!< refinement info
539:     int                       _nref;  //!< number of regular refinements performed
540:     std::vector<int>          _vfathers; //!< stores the 2 fathers of each vertex (e
qual fathers denote original coarse vertex)
541:
542: };
543:
544: // ****************************************************************************
545:
546: class gMesh_Hierarchy
547: {
548: public:
549:     /**
550:      * Constructs mesh hierarchy of @p nlevel levels starting with coarse mesh @p cm
esh.
551:      * The coarse mesh @p cmesh will be @p nlevel-1 times geometrically refined.
552:      *
553:      * @param[in] cmesh   initial coarse mesh
554:      * @param[in] nlevel  number levels in mesh hierarchy
555:      *
556:      */
557:     gMesh_Hierarchy(Mesh const &cmesh, int nlevel);
558:
559:     size_t size() const
560:     {
561:         return _gmesh.size();
562:     }
563:
564:     /**
565:      * Access to mesh @p lev from mesh hierarchy.
566:      *
567:      * @return mesh @p lev
568:      * @warning An out_of_range exception might be thrown.
569:      *
570:      */
571:     Mesh const &operator[](int lev) const
572:     {
573:         return *_gmesh.at(lev);
574:     }
575:
576:     /**
577:      * Access to finest mesh in mesh hierarchy.
578:      *
579:      * @return finest mesh
580:      *
581:      */
582:     Mesh const &finest() const
583:     {
584:         return *_gmesh.back();
585:     }
586:
587:     /**
588:      * Access to coarest mesh in mesh hierarchy.
```

```
  589:       *
  590:       * @return coarsest mesh
  591:       *
  592:       */
  593:      Mesh const &coarsest() const
  594:      {
  595:          return *_gmesh.front();
  596:      }
  597:
  598: private:
  599:      std::vector<std::shared_ptr<Mesh>> _gmesh; //!< mesh hierarchy from coarse ([0])
 to fine.
  600:
  601: };
  602:
  603:
  604:
  605: // ****************************************************************
  606: /**
  607:  * 2D finite element mesh of the square consisting of linear triangular elements.
  608:  */
  609: class Mesh_2d_3_square: public Mesh
  610: {
  611: public:
  612:      /**
  613:       * Generates the f.e. mesh for the unit square.
  614:       *
  615:       * @param[in] nx    number of discretization intervals in x-direction
  616:       * @param[in] ny    number of discretization intervals in y-direction
  617:       * @param[in] myid  my MPI-rank / subdomain
  618:       * @param[in] procx number of ranks/subdomains in x-direction
  619:       * @param[in] procy number of processes in y-direction
  620:      */
  621:      Mesh_2d_3_square(int nx, int ny, int myid = 0, int procx = 1, int procy = 1);
  622:
  623:      /**
  624:       * Destructor
  625:       */
  626:      ~Mesh_2d_3_square() override;
  627:
  628:      /**
  629:       * Set solution vector based on a tensor product grid in the rectangle.
  630:       * @param[in] u solution vector
  631:       */
  632:      void SetU(std::vector<double> &u) const;
  633:
  634:      /**
  635:       * Set right hand side (rhs) vector on a tensor product grid in the rectangle.
  636:       * @param[in] f rhs vector
  637:       */
  638:      void SetF(std::vector<double> &f) const;
  639:
  640:      /**
  641:       * Determines the indices of those vertices with Dirichlet boundary conditions
  642:       * @return index vector.
  643:       */
  644:      std::vector<int> Index_DirichletNodes() const override;
  645:      std::vector<int> Index_BoundaryNodes() const override;
  646:
  647:      /**
  648:        * Stores the values of vector @p u of (sub)domain into a file @p name for furt
her processing in gnuplot.
  649:        * The file stores rowise the x- and y- coordinates together with the value fro
m  @p u .
  650:        * The domain [@p xl, @p xr] x [@p yb, @p yt] is discretized into @p nx x @p ny
 intervals.
  651:        *
  652:        * @param[in] name  basename of file name (file name will be extended by the ra
```

```
nk number)
  653:       * @param[in] u     local vector
  654:       *
  655:       * @warning   Assumes tensor product grid in unit square; rowise numbered
  656:       *            (as generated in class constructor).
  657:       *            The output is provided for tensor product grid visualization
  658:       *            ( similar to Matlab-surf() ).
  659:       *
  660:       * @see Mesh_2d_3_square
  661:       */
  662:     void SaveVectorP(std::string const &name, std::vector<double> const &u) const;
  663:
  664:     // here will still need to implement in the class
  665:     //  GetBound(), AddBound()
  666:     //  or better a generalized way with indices and their appropriate ranks for MPI
 communication
  667:
  668: private:
  669:     /**
  670:       * Determines the coordinates of the discretization nodes of the domain [@p xl,
 @p xr] x [@p yb, @p yt]
  671:       * which is discretized into @p nx x @p ny intervals.
  672:       * @param[in] nx    number of discretization intervals in x-direction
  673:       * @param[in] ny    number of discretization intervals in y-direction
  674:       * @param[in] xl    x-coordinate of left boundary
  675:       * @param[in] xr    x-coordinate of right boundary
  676:       * @param[in] yb    y-coordinate of lower boundary
  677:       * @param[in] yt    y-coordinate of upper boundary
  678:       * @param[out] xc   coordinate vector of length 2n with x(2*k,2*k+1) as coordin
ates of node k
  679:       */
  680:
  681:     void GetCoordsInRectangle(int nx, int ny, double xl, double xr, double yb, doubl
e yt,
  682:                               double xc[]);
  683:     /**
  684:       * Determines the element connectivity of linear triangular elements of a FEM d
iscretization
  685:       * of a rectangle using @p nx x @p ny equidistant intervals for discretization.
  686:       * @param[in] nx    number of discretization intervals in x-direction
  687:       * @param[in] ny    number of discretization intervals in y-direction
  688:       * @param[out] ia   element connectivity matrix with ia(3*s,3*s+1,3*s+2) as nod
e numbers od element s
  689:       */
  690:     void GetConnectivityInRectangle(int nx, int ny, int ia[]);
  691:
  692: private:
  693:     int _myid;          //!< my MPI rank
  694:     int _procx;         //!< number of MPI ranks in x-direction
  695:     int _procy;         //!< number of MPI ranks in y-direction
  696:     std::array<int, 4> _neigh; //!< MPI ranks of neighbors (negative: no neighbor bu
t b.c.)
  697:     int _color;         //!< red/black coloring (checker board) of subdomains
  698:
  699:     double _xl;         //!< x coordinate of lower left  corner of square
  700:     double _xr;         //!< x coordinate of lower right corner of square
  701:     double _yb;         //!< y coordinate or lower left  corner of square
  702:     double _yt;         //!< y coordinate of upper right corner of square
  703:     int    _nx;         //!< number of intervals in x-direction
  704:     int    _ny;         //!< number of intervals in y-direction
  705: };
  706:
  707: // ****************************************************************
  708:
  709:
  710:
  711:
  712: #endif
```

```
    1: //              MPI code in C++.
    2: //              See [Gropp/Lusk/Skjellum, "Using MPI", p.33/41 etc.]
    3: //              and  /opt/mpich/include/mpi2c++/comm.h  for details
    4:
    5: #include "geom.h"
    6: #include "par_geom.h"
    7: #include "vdop.h"
    8:
    9: #include <cassert>
   10: #include <cmath>
   11: #include <iostream>
   12: #include <mpi.h>             // MPI
   13: #include <omp.h>             // OpenMP
   14: using namespace std;
   15:
   16:
   17: int main(int argc, char **argv )
   18: {
   19:     MPI_Init(&argc, &argv);
   20:     MPI_Comm const icomm(MPI_COMM_WORLD);
   21:     omp_set_num_threads(1);                    // don't use OMP parallelization for a
 start
   22: //
   23:    {
   24:        int np;
   25:        MPI_Comm_size(icomm, &np);
   26:
   27:     //    assert(4 == np);                  // example is only provided for 4 MPI pr
ocesses
   28:    }
   29: // ######################################################################
   30: // ---- Read the f.e. mesh and the mapping of elements to MPI processes
   31:     //Mesh const mesh_c("square_4.txt");    //    Files square_4.txt and square_4_sd
.txt  are needed
   32:     ParMesh const mesh("square",icomm);
   33:
   34:     int const numprocs = mesh.NumProcs();
   35:     int const myrank   = mesh.MyRank();
   36:     if ( 0 == myrank ) {
   37:         cout << "\n There are " << numprocs << " processes running.\n \n";
   38:     }
   39:
   40:     int const check_rank=1;               // choose the MPI process you would like t
o check the mesh
   41:     //if ( check_rank == myrank ) mesh.Debug();
   42:     //if ( check_rank == myrank ) mesh.DebugEdgeBased();
   43:
   44:
   45: // ######################################################################
   46: // ---- allocate local vectors and check skalar product and vector accumulation
   47:
   48:     if (check_rank==myrank) {printf("\n\n-------------- Task 9 --------------\n\n");
}
   49:     if (check_rank==myrank) cout << "Mesh coordinates: " << mesh.GetCoords() << endl
 << endl;
   50:     MPI_Barrier(icomm);
   51:     vector<double> xl(mesh.Nnodes(), 1.0);
   52:
   53:     // for visualization I had to type in terminal:
   54:     // export LIBGL_ALWAYS_SOFTWARE=1
   55:     if (check_rank==myrank) mesh.Visualize(xl);
   56:
   57:     double ss = mesh.dscapr(xl,xl);
   58:     cout << myrank << " : scalar : " << ss << endl;
   59:
   60:     mesh.VecAccu(xl);
   61:     if (check_rank==myrank) mesh.Visualize(xl);
   62:
```

```
    63:
    64:     MPI_Barrier(icomm);
    65:     if (check_rank==myrank) {printf("\n\n-------------- Task 10 --------------\n\n")
;}
    66:     vector<int> y(mesh.Nnodes(), 1);
    67:     mesh.VecAccuInt(y);
    68:     if (check_rank==myrank) {
    69:         printf("Accumulated integer vector y:\n");
    70:         for (int i : y) {
    71:             cout << i << " ";
    72:         }
    73:     }
    74:
    75:
    76:     MPI_Barrier(icomm);
    77:     if (check_rank==myrank) {printf("\n\n-------------- Task 11 --------------\n\n")
;}
    78:     int global_nodes = mesh.GlobalNodes();
    79:     if (check_rank==myrank) cout << "Global nodes: " << global_nodes << endl;
    80:
    81:
    82:     MPI_Barrier(icomm);
    83:     if (check_rank==myrank) {printf("\n\n-------------- Task 12 --------------\n\n")
;}
    84:
    85:     // Set xl to 1s vector again
    86:     for (size_t k=0; k<xl.size(); ++k)
    87:     {
    88:         xl[k] = 1.0;
    89:     }
    90:     if (check_rank==myrank) mesh.Visualize(xl);
    91:     mesh.Average(xl);
    92:     if (check_rank==myrank) mesh.Visualize(xl);
    93:
    94:
    95:     // -------------- Task 13 --------------
    96:     // Should work with 2, 4 and 6 subdomains (change run target in GCC_default.mk)
    97:     // Check subdomains with different values for check_rank (0-5)
    98:
    99:
   100:
   101:     MPI_Finalize();
   102:     return 0;
   103: }
   104:
   105:
```

```
    1: // see:   http://llvm.org/docs/CodingStandards.html#include-style
    2: #include "vdop.h"
    3: //#include "geom.h"
    4: #include "par_geom.h"
    5:
    6: #include <algorithm>
    7: #include <array>
    8: #include <cassert>
    9: #include <cmath>
   10: #include <ctime>                  // contains clock()
   11: #include <fstream>
   12: #include <iostream>
   13: #include <list>
   14: #include <numeric>                // accumulate()
   15: #include <string>
   16: #include <vector>
   17:
   18: using namespace std;
   19:
   20:
   21: ParMesh::ParMesh(int ndim, int nvert_e, int ndof_e, int nedge_e, MPI_Comm const &ico
mm)
   22:     : Mesh(ndim, nvert_e, ndof_e, nedge_e),
   23:       _icomm(icomm), _numprocs(-1), _myrank(-1),
   24:       _v_l2g(0), _t_l2g(0), _v_g2l{{}}, _t_g2l{{}}, _valence(0),
   25:       _sendbuf(0), _sendcounts(0), _sdispls(0),
   26:       _loc_itf(0), _gloc_itf(0), _buf2loc(0)
   27: {
   28:     MPI_Comm_size(icomm, &_numprocs);
   29:     MPI_Comm_rank(icomm, &_myrank);
   30: }
   31:
   32: ParMesh::~ParMesh()
   33: {}
   34:
   35:
   36:
   37: ParMesh::ParMesh(std::string const &sname, MPI_Comm const &icomm)
   38:     : ParMesh(2, 3, 3, 3, icomm) // two dimensions, 3 vertices, 3 dofs, 3 edges per
element
   39: {
   40:     //const int numprocs = _icomm.Get_size();
   41:     const string NS    = "_" + to_string(_numprocs);
   42:     const string fname = sname + NS + ".txt";
   43:     //cout << "############  " << fname << endl;
   44:     ReadVertexBasedMesh(fname);
   45:     cout << "\n End of sequential File read \n";
   46:     // ------------------------------------------------------------------------
-
   47:     // Until this point  a l l  processes possess  a l l  mesh info in  g l o b a l
 numbering
   48:     //
   49:     // Now, we have to select the data belonging to my_rank
   50:     // and we have to create the mapping local to global (l2g) and vice versa (g2l)
   51:     // ------------------------------------------------------------------------
-
   52:
   53:     // save the global node mesh (maybe we need it later)
   54:     DeriveEdgeFromVertexBased();                    //                    and even
more
   55:     Mesh global_mesh(*this);                        // requires a  l o t  of memory
   56:     Del_EdgeConnectivity();
   57:
   58:     // read the subdomain info
   59:     const string dname = sname + NS + "_sd" + ".txt";
   60:     vector<int> t2d = ReadElementSubdomains(dname);  // global mapping triangle to s
ubdomain for all elements
   61:
```

```
   62:     //const int myrank   = _icomm.Get_rank();
   63:     Transform_Local2Global_Vertex(_myrank, t2d);      // Vertex based mesh: now in
l o c a l  indexing
   64:
   65:     DeriveEdgeFromVertexBased();                     // Generate also the  l o c a l
  edge based information
   66:
   67:     Generate_VectorAdd();
   68:
   69:
   70:     // Now we have to organize the MPI communication of vertices on the subdomain in
terfaces
   71:
   72:     return;
   73: }
   74:
   75: vector<int> ParMesh::ReadElementSubdomains(string const &dname)
   76: {
   77:     ifstream ifs(dname);
   78:     if (!(ifs.is_open() && ifs.good())) {
   79:         cerr << "ParMesh::ReadElementSubdomain: Error cannot open file " << dname <<
 endl;
   80:         assert(ifs.is_open());
   81:     }
   82:
   83:     int const OFFSET{1};             // Matlab to C indexing
   84:     cout << "ASCI file  " << dname << "  opened" << endl;
   85:
   86:     // Read some mesh constants
   87:     int nelem;
   88:     ifs >> nelem;
   89:     cout << nelem << "  " << Nelems() << endl;
   90:     assert( Nelems() == nelem);
   91:
   92:     // Allocate memory
   93:     vector<int> t2d(nelem, -1);
   94:     // Read element mapping
   95:     for (int k = 0; k < nelem; ++k) {
   96:         int tmp;
   97:         ifs >> tmp;
   98:         //t2d[k] = tmp - OFFSET;
   99:         // 2020-01-08
  100:         t2d[k] = min(tmp, NumProcs()) - OFFSET;
  101:     }
  102:
  103:     return t2d;
  104: }
  105:
  106: void ParMesh::Transform_Local2Global_Vertex(int const myrank, vector<int> const &t2d
)
  107: {
  108:     // number of local elements
  109:     const int l_ne = count(t2d.cbegin(), t2d.cend(), myrank);
  110:     //cout << myrank <<  "::  " << lne << endl;
  111:     vector<int> l_ia(l_ne * NverticesElements(), -1); // local elements still with g
lobal vertex numbers
  112:     _t_l2g.resize(l_ne, -1);
  113:
  114:     int lk = 0;
  115:     for (size_t k = 0; k < t2d.size(); ++k) {
  116:         if (myrank == t2d[k]) {
  117:             //if (0==myrank)
  118:             //{
  119:             //cout << lk << "   k  " <<  t2d[k] << endl;
  120:             //}
  121:             l_ia[3 * lk  ] = _ia[3 * k  ];
  122:             l_ia[3 * lk + 1] = _ia[3 * k + 1];
  123:             l_ia[3 * lk + 2] = _ia[3 * k + 2]; // local elements still with global v
```

```
ertex numbers
  124:             _t_l2g[lk] = k;                         // elements: local to global mappi
ng
  125:             _t_g2l[k] = lk;                         //           global to local
  126:             ++lk;
  127:         }
  128:     }
  129:     // Checks:
  130:     assert( count(l_ia.cbegin(), l_ia.cend(), -1)   == 0 );
  131:     assert( count(_t_l2g.cbegin(), _t_l2g.cend(), -1) == 0 );
  132:
  133:     // Vertices: local to global mapping
  134:     auto tmp = l_ia;
  135:     sort(tmp.begin(), tmp.end());
  136:     auto ip = unique(tmp.begin(), tmp.end());
  137:     tmp.erase(ip, tmp.end());
  138:     _v_l2g = tmp;                                   // Vertices: local to global mappi
ng
  139:     for (size_t lkv = 0; lkv < _v_l2g.size(); ++lkv) {
  140:         _v_g2l[_v_l2g[lkv]] = lkv;                  //           global to local
  141:     }
  142:
  143:     // Boundary edges
  144:     vector<int> l_bedges;
  145:     vector<int> l_sdedges;
  146:     for (size_t b = 0; b < _bedges.size(); b += 2) {
  147:         int const v1 = _bedges[b   ];               // global vertex numbers
  148:         int const v2 = _bedges[b + 1];
  149:         try {
  150:             int const lv1 = _v_g2l.at(v1);          // map[] would add that element
  151:             int const lv2 = _v_g2l.at(v2);          //       but at() throws an exept
ion
  152:             l_bedges.push_back(lv1);
  153:             l_bedges.push_back(lv2);                // Boundaries: already in local i
ndexing
  154:             // 2020-01-08
  155:             l_sdedges.push_back(_sdedges[b  ]);
  156:             l_sdedges.push_back(_sdedges[b+1]);
  157:         }
  158:         catch (std::out_of_range & err) {
  159:             //cerr << ".";
  160:         }
  161:     }
  162:
  163:     // number of local vertices
  164:     const int l_nn = _v_l2g.size();
  165:     vector<double> l_xc(Ndims()*l_nn);
  166:     for (int lkk = 0; lkk < l_nn; ++lkk) {
  167:         int k = _v_l2g.at(lkk);
  168:         l_xc[2 * lkk  ]   = _xc[2 * k  ];
  169:         l_xc[2 * lkk + 1] = _xc[2 * k + 1];
  170:     }
  171:
  172:
  173:     // Now, we represent the vertex mesh in  l o c a l  numbering
  174:     // elements
  175:
  176:     for (size_t i = 0; i < l_ia.size(); ++i) {
  177:         l_ia[i] = _v_g2l.at(l_ia[i]);               // element vertices: global to lo
cal
  178:     }
  179:     SetNelem(l_ne);
  180:     _ia = l_ia;
  181:     // boundary
  182:     _bedges = l_bedges;
  183:     _sdedges = l_sdedges;
  184:     // coordinates
  185:     SetNnode(l_nn);
```

```
  186:     _xc = l_xc;
  187:
  188:     return;
  189: }
  190:
  191:
  192: void ParMesh::Generate_VectorAdd()
  193: {
  194:     // Some checks
  195:     int lnn = Nnodes();                              // local number of vertices
  196:     assert(static_cast<int>(_v_l2g.size()) == lnn);
  197:     int ierr{-12345};
  198:
  199:     // ---- Determine global largest vertex index
  200:     int gidx_max{-1};                                // global largest vertex index
  201:     int lmax = *max_element(_v_l2g.cbegin(), _v_l2g.cend());
  202:     MPI_Allreduce(&lmax, &gidx_max, 1, MPI_INT, MPI_MAX, _icomm);
  203:     int gidx_min{-1};                                // global smallest vertex index
  204:     int lmin = *min_element(_v_l2g.cbegin(), _v_l2g.cend());
  205:     MPI_Allreduce(&lmin, &gidx_min, 1, MPI_INT, MPI_MIN, _icomm);
  206:     //cout << gidx_min << "  " << gidx_max << endl;
  207:     assert(0 == gidx_min);                           // global indices have to start wi
th 0
  208:
  209:
  210:     // ---- Determine for all global vertices the number of subdomains it belongs to
  211:     vector<int> global(gidx_max+1, 0);               // global scalar array for verti
ces
  212:     for (auto const gidx : _v_l2g)  global[gidx] = 1;
  213:     // https://www.mpi-forum.org/docs/mpi-2.2/mpi22-report/node109.htm
  214:     ierr = MPI_Allreduce(MPI_IN_PLACE, global.data(), global.size(), MPI_INT, MPI_SU
M, _icomm);
  215:     //if (0 == MyRank())  cout << global << endl;
  216:     //MPI_Barrier(_icomm);
  217:     //cout << _xc[2*_v_g2l.at(2)] << " , " << _xc[2*_v_g2l.at(2)+1] << endl;
  218:     //MPI_Barrier(_icomm);
  219:
  220:     //  now, global[] contains the number of subdomains a global vertex belongs to
  221:     if ( count(global.cbegin(), global.cend(), 0) > 0 )
  222:         cerr << "\n !!!   Non-continuous global vertex indexing  !!!\n";
  223:
  224:     // ---- Determine local interface vertices ( <==> global[] > 1 )
  225:     //      _loc_itf,  neigh_itf
  226:     //vector<int> loc_itf;                           // local indices of interface ve
rtices on this MPI process
  227:     for (size_t lk = 0; lk < _v_l2g.size(); ++lk) {
  228:         int const gk = _v_l2g[lk];                   // global index of local vertex lk
  229:         if ( global[gk] > 1 ) {
  230:             _loc_itf.push_back(lk);                  // local indices of interface vert
ices on this MPI process
  231:         }
  232:     }
  233:
  234:     //MPI_Barrier(_icomm);
  235:     //if (0 == MyRank())  cout << "\n..._loc_itf...\n" << _loc_itf << "\n......\n";
  236:     //MPI_Barrier(_icomm);
  237:     // ---- global indices of local interface vertices
  238:     //auto gloc_itf(_loc_itf);
  239:     _gloc_itf=_loc_itf;
  240:     for_each(_gloc_itf.begin(), _gloc_itf.end(), [this] (auto & v) -> void { v = _v_
l2g[v];} );
  241:     //MPI_Barrier(_icomm);
  242:     //if (0 == MyRank())  cout << "\n..._gloc_itf...\n" << _gloc_itf << "\n......\n"
;
  243:     //DebugVector(_gloc_itf,"_gloc_itf");
  244:
  245:     // ---- Determine the global length of interfaces
  246:     vector<int> vnn(NumProcs(), -1);                 // number of interface vertices pe
```

```
r MPI rank
  247:     int l_itf(_loc_itf.size());                    // # local interface vertices
  248:     ierr = MPI_Allgather(&l_itf, 1, MPI_INT, vnn.data(), 1, MPI_INT, _icomm);
  249:     assert(0 == ierr);
  250:     //cout << vnn << endl;
  251:
  252:     // ---- Now we consider only the inferface vertices
  253:     int snn = accumulate(vnn.cbegin(), vnn.cend(), 0); // required length of array f
or global interface indices
  254:     //cout << snn << "   " << gnn << endl;
  255:     vector<int> dispnn(NumProcs(), 0) ;          // displacement of interface verti
ces per MPI rank
  256:     partial_sum(vnn.cbegin(), vnn.cend() - 1, dispnn.begin() + 1);
  257:     //cout << dispnn << endl;
  258:
  259:     // ---- Get the global indices for all global interfaces
  260:     vector<int> g_itf(snn, -1);                    // collects all global indices of
 the global interfaces
  261:     // https://www.mpich.org/static//docs/v3.0.x/www3/MPI_Gatherv.html
  262:     ierr = MPI_Gatherv( _gloc_itf.data(), _gloc_itf.size(), MPI_INT,
  263:                         g_itf.data(), vnn.data(), dispnn.data(), MPI_INT, 0, _icomm)
;
  264:     assert(0 == ierr);
  265:     // https://www.mpich.org/static/docs/v3.1/www3/MPI_Bcast.html
  266:     ierr = MPI_Bcast(g_itf.data(), g_itf.size(), MPI_INT, 0, _icomm);
  267:     assert(0 == ierr);                             // Now, each MPI rank has the all
global indices of the global interfaces
  268:     //MPI_Barrier(_icomm);
  269:     //if (MyRank() == 0) cout << "\n...g_itf...\n" << g_itf << "\n......\n";
  270:     //MPI_Barrier(_icomm);
  271:
  272:     // ----- Determine all MPI ranks a local interface vertex belongs to
  273:     vector<vector<int>> neigh_itf(_loc_itf.size());// subdomains a local interface v
ertex belongs to
  274:     for (size_t lk = 0; lk < _loc_itf.size(); ++lk) {
  275:         const int gvert = _gloc_itf[lk];           // global index of local interfac
e node lk
  276:         for (int rank = 0; rank < NumProcs(); ++rank) {
  277:             auto const startl = g_itf.cbegin() + dispnn[rank];
  278:             auto const endl   = startl + vnn[rank];
  279:             if ( find( startl, endl, gvert) != endl) {
  280:                 neigh_itf[lk].push_back(rank);
  281:             }
  282:         }
  283:     }
  284:
  285:     // ---- check the available info in  _loc_itf[lk], _gloc_itf[lk], neigh_itf[lk]
  286:     //MPI_Barrier(_icomm);
  287:     ////if (MyRank()==0)  cout << "\n...neigh_itf ...\n"  << neigh_itf << endl;
  288:     //if (MyRank() == 0) {
  289:         //for (size_t lk = 0; lk < _loc_itf.size(); ++lk ) {
  290:             //cout << lk << " : local idx " << _loc_itf[lk] << " , global idx " <<
_gloc_itf[lk];
  291:             //cout << "  with MPI ranks " << neigh_itf[lk] << endl;
  292:         //}
  293:     //}
  294:     //MPI_Barrier(_icomm);
  295:
  296:     // ---- store the valence (e.g., the number of subdomains it belongs to) of all
local vertices
  297:     _valence.resize(Nnodes(),1);
  298:     for (size_t lk = 0; lk < _loc_itf.size(); ++lk)
  299:     {
  300:         _valence[_loc_itf[lk]] = neigh_itf[lk].size();
  301:     }
  302:     //DebugVector(_valence,"_valence",_icomm);
  303:
  304:     // ---- We ware going to use MPI_Alltoallv for data exchange on interfaces
```

```
  305:      //      https://www.mpi-forum.org/docs/mpi-3.1/mpi31-report/node109.htm#Node109
  306:      //      https://www.open-mpi.org/doc/v4.0/man3/MPI_Alltoallv.3.php
  307:      //int MPI_Alltoallv(const void* sendbuf, const int sendcounts[], const int sdisp
ls[], MPI_Datatype sendtype, void* recvbuf, const int recvcounts[], const int rdispls[], MP
I_Datatype recvtype, MPI_Comm comm)
  308:      //
  309:      //  MPI_Alltoallv needs:
  310:      //      vector<double> sendbuf            (MPI_IN_PLACE: used also as recvbuf)
  311:      //      vector<int> sendcounts            (the same as for recv)
  312:      //      vector<int> sdispls               (the same as for recv)
  313:      //
  314:      //  We need to map the interface vertices onto the sendbuffer:
  315:      //      vector<int> loc_itf                local  index of interface vertex lk
  316:      //      vector<int> gloc_itf               global index of interface vertex lk
  317:      //      vector<int> buf2loc                local indices of sendbuffer position
s (the same as for recv)
  318:
  319:      // ---- Determine sendcounts[] and sdipls[] from neigh_itf[]
  320:      //vector<int> _sendcounts(NumProcs(), 0);
  321:      _sendcounts.resize(NumProcs(), 0);
  322:      for (size_t lk = 0; lk < _loc_itf.size(); ++lk ) {
  323:          auto const &kneigh = neigh_itf[lk];
  324:          for (size_t ns = 0; ns < kneigh.size(); ++ns) {
  325:              ++_sendcounts[kneigh[ns]];
  326:          }
  327:      }
  328:      //if (MyRank() == 0)  cout << "\n..._sendcounts ...\n"  << _sendcounts << endl;
  329:
  330:      //vector<int> _sdispls(NumProcs(), 0);
  331:      _sdispls.resize(NumProcs(), 0);
  332:      partial_sum(_sendcounts.cbegin(), _sendcounts.cend() - 1, _sdispls.begin() + 1);
  333:      //vector<int> _sdispls(NumProcs()+1, 0);
  334:      //partial_sum(_sendcounts.cbegin(), _sendcounts.cend(), _sdispls.begin() + 1);
  335:      //if (MyRank() == 0)  cout << "\n..._sdispls ...\n"  << _sdispls << endl;
  336:
  337:      // ---- Determine size of buffer 'nbuffer' and mapping 'buf2loc'
  338:      int const nbuffer = accumulate(_sendcounts.cbegin(), _sendcounts.cend(), 0);
  339:      //vector<int> _buf2loc(nbuffer, -1);
  340:      _buf2loc.resize(nbuffer, -1);
  341:      int buf_idx = 0;                                  // position in buffer
  342:      for (int rank = 0; rank < NumProcs(); ++rank) {
  343:          assert( buf_idx ==  _sdispls[rank]);
  344:          for (size_t lk = 0; lk < _loc_itf.size(); ++lk ) {
  345:              auto const &kneigh = neigh_itf[lk];
  346:              if (find(kneigh.cbegin(),kneigh.cend(),rank)!=kneigh.cend())
  347:              {
  348:                 _buf2loc[buf_idx] =  _loc_itf[lk];
  349:                 ++buf_idx;
  350:              }
  351:          }
  352:      }
  353:      //if (MyRank() == 0)  cout << "\n...buf2loc ...\n"  << buf2loc << endl;
  354:      //DebugVector(buf2loc,"buf2loc",_icomm);
  355:
  356:      // ---- Allocate send/recv buffer
  357:      //vector<double> _sendbuf(nbuffer,-1.0);
  358:      _sendbuf.resize(nbuffer,-1.0);
  359:
  360:      assert(CheckInterfaceExchange_InPlace());
  361:      cout << " Check of data exchange (InPlace) successful!\n";
  362:      assert(CheckInterfaceExchange());
  363:      cout << " Check of data exchange successful!\n";
  364:      assert(CheckInterfaceAdd_InPlace());
  365:      cout << " Check of data add successful!\n";
  366:      assert(CheckInterfaceAdd());
  367:      cout << " Check of data add (InPlace) successful!\n";
  368:
  369:      vector<double> x(Nnodes(),-1.0);
```

```
  370:     VecAccu(x);
  371:     cout << " VecAccu (InPlace) successful!\n";
  372:
  373:
  374:     return;
  375: }
  376:
  377: bool ParMesh::CheckInterfaceExchange_InPlace() const
  378: {
  379:     vector<double> x(Nnodes(),-1.0);
  380:     copy(_v_l2g.cbegin(),_v_l2g.cend(),x.begin());       // init x with global verte
x indices
  381:
  382:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  383:     {
  384:         _sendbuf[ls] = x[_buf2loc.at(ls)];
  385:     }
  386:     int ierr = MPI_Alltoallv(MPI_IN_PLACE, _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE,
  387:                              _sendbuf.data(), _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE, _icomm);
  388:     assert(ierr==0);
  389:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  390:
  391:     vector<double> y(x);
  392:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) y[_loc_itf.at(lk)] = -1.0;   // onl
y for interface nodes
  393:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  394:     {
  395:         y[_buf2loc.at(ls)] = _sendbuf[ls];
  396:     }
  397:
  398:     double const  eps=1e-10;
  399:     bool bv = equal(x.cbegin(),x.cend(),y.cbegin(),
  400:                             [eps](double a, double b) -> bool
  401:                             { return std::abs(a-b)<eps*(1.0+0.5*(std::abs(a)+ std::abs
(b))); }
  402:                     );
  403:     return bv;
  404: }
  405:
  406: bool ParMesh::CheckInterfaceExchange() const
  407: {
  408:     vector<double> x(Nnodes(),-1.0);
  409:     copy(_v_l2g.cbegin(),_v_l2g.cend(),x.begin());       // init x with global verte
x indices
  410:
  411:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  412:     {
  413:         _sendbuf[ls] = x[_buf2loc.at(ls)];
  414:     }
  415:     vector<double> recvbuf(_sendbuf.size());
  416:     int ierr = MPI_Alltoallv(_sendbuf.data(), _sendcounts.data(), _sdispls.data(), M
PI_DOUBLE,
  417:                              recvbuf.data(), _sendcounts.data(), _sdispls.data(), M
PI_DOUBLE, _icomm);
  418:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  419:     //DebugVector(recvbuf,"recvbuf",_icomm);
  420:     assert(ierr==0);
  421:
  422:     vector<double> y(x);
  423:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) y[_loc_itf.at(lk)] = -1.0;   // onl
y for interface nodes
  424:     for(size_t ls = 0; ls<recvbuf.size(); ++ls)
  425:     {
  426:         y[_buf2loc.at(ls)] = recvbuf[ls];
  427:     }
  428:     //cout << "WRONG  : " << count(y.cbegin(),y.cend(), -1.0) << endl;
```

```
  429:
  430:     double const  eps=1e-10;
  431:     bool bv = equal(x.cbegin(),x.cend(),y.cbegin(),
  432:                         [eps](double a, double b) -> bool
  433:                         { return std::abs(a-b)<eps*(1.0+0.5*(std::abs(a)+ std::abs
(b))); }
  434:                     );
  435:     return bv;
  436: }
  437:
  438: bool ParMesh::CheckInterfaceAdd_InPlace() const
  439: {
  440:     vector<double> x(Nnodes(),-1.0);
  441:     for (size_t i=0; i<x.size(); ++i)
  442:     {
  443:         x[i] = _xc[2*i]+_xc[2*i+1];                     // init x with coordinate v
alues
  444:     }
  445:
  446:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  447:     {
  448:         _sendbuf[ls] = x[_buf2loc.at(ls)];
  449:     }
  450:     int ierr = MPI_Alltoallv(MPI_IN_PLACE, _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE,
  451:                         _sendbuf.data(), _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE, _icomm);
  452:     assert(ierr==0);
  453:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  454:
  455:     vector<double> y(x);
  456:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) y[_loc_itf.at(lk)] = 0.0;   // only
 for interface nodes
  457:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  458:     {
  459:         y[_buf2loc.at(ls)] += _sendbuf[ls];
  460:     }
  461:     MPI_Barrier(_icomm);
  462:     //DebugVector(x,"x",_icomm);
  463:     //DebugVector(y,"y",_icomm);
  464:     for (size_t i= 0; i<y.size(); ++i) y[i]/=_valence[i];   // divide by valence
  465:
  466:     double const  eps=1e-10;
  467:     bool bv = equal(x.cbegin(),x.cend(),y.cbegin(),
  468:                         [eps](double a, double b) -> bool
  469:                         { return std::abs(a-b)<eps*(1.0+0.5*(std::abs(a)+ std::abs
(b))); }
  470:                     );
  471:     return bv;
  472: }
  473:
  474: bool ParMesh::CheckInterfaceAdd() const
  475: {
  476:     vector<double> x(Nnodes(),-1.0);
  477:     for (size_t i=0; i<x.size(); ++i)
  478:     {
  479:         //x[i] = _xc[2*i]+_xc[2*i+1];                   // init x with coordinate
 values
  480:         x[i] = _v_l2g[i];
  481:     }
  482:
  483:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  484:     {
  485:         _sendbuf[ls] = x[_buf2loc.at(ls)];
  486:     }
  487:     vector<double> recvbuf(_sendbuf.size());
  488:     int ierr = MPI_Alltoallv(_sendbuf.data(), _sendcounts.data(), _sdispls.data(), M
PI_DOUBLE,
```

```
  489:                                 recvbuf.data(), _sendcounts.data(), _sdispls.data(), M
PI_DOUBLE, _icomm);
  490:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  491:     //DebugVector(recvbuf,"recvbuf",_icomm);
  492:     assert(ierr==0);
  493:
  494:     vector<double> y(x);
  495:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) y[_loc_itf.at(lk)] = 0.0;   // only
 for interface nodes
  496:     for(size_t ls = 0; ls<recvbuf.size(); ++ls)
  497:     {
  498:         //if (0==MyRank()) cout << ls << ": " << _buf2loc.at(ls) << "  " << y[_buf2l
oc.at(ls)] << "("<< x[_buf2loc.at(ls)] << ")" << "  " << recvbuf[ls] << "  (" <<  _sendbuf[
ls] << ")" << endl;
  499:         y[_buf2loc.at(ls)] += recvbuf[ls];
  500:     }
  501:     MPI_Barrier(_icomm);
  502:     //DebugVector(x,"x",_icomm);
  503:     //DebugVector(y,"y",_icomm);
  504:     for (size_t i= 0; i<y.size(); ++i) y[i]/=_valence[i];   // divide by valence
  505:
  506:     double const  eps=1e-10;
  507:     bool bv = equal(x.cbegin(),x.cend(),y.cbegin(),
  508:                             [eps](double a, double b) -> bool
  509:                             { return std::abs(a-b)<eps*(1.0+0.5*(std::abs(a)+ std::abs
(b))); }
  510:                     );
  511:     return bv;
  512: }
  513:
  514:
  515: // ----------
  516:
  517: void ParMesh::VecAccu(std::vector<double> &w) const
  518: {
  519:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  520:     {
  521:         _sendbuf[ls] = w[_buf2loc.at(ls)];
  522:     }
  523:     int ierr = MPI_Alltoallv(MPI_IN_PLACE, _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE,
  524:                             _sendbuf.data(), _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE, _icomm);
  525:     assert(ierr==0);
  526:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  527:
  528:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) w[_loc_itf.at(lk)] = 0.0;   // only
 for interface nodes
  529:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  530:     {
  531:         w[_buf2loc.at(ls)] += _sendbuf[ls];
  532:     }
  533:
  534:     return;
  535: }
  536:
  537:
  538: // ############################################################################
  539: // ############################################################################
  540:
  541:
  542: // ---- EX10 ----
  543: void ParMesh::VecAccuInt(std::vector<int> &w) const
  544: {
  545:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  546:     {
  547:         _sendbuf[ls] = w[_buf2loc.at(ls)];
  548:     }
```

```
  549:     int ierr = MPI_Alltoallv(MPI_IN_PLACE, _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE,
  550:                              _sendbuf.data(), _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE, _icomm);
  551:     assert(ierr==0);
  552:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  553:
  554:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) w[_loc_itf.at(lk)] = 0.0;   // only
 for interface nodes
  555:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  556:     {
  557:         w[_buf2loc.at(ls)] += _sendbuf[ls];
  558:     }
  559:
  560:     return;
  561: }
  562:
  563:
  564: // ---- EX11 ----
  565: int ParMesh::GlobalNodes() const
  566: {
  567:     int local_count = 0;
  568:     for (int i=0; i<Nnodes(); ++i) {
  569:         local_count += 1.0 / _valence[i];
  570:     }
  571:
  572:     int global_nodes = 0;
  573:     MPI_Allreduce(&local_count, &global_nodes, 1, MPI_INT, MPI_SUM, _icomm);
  574:
  575:     return global_nodes;
  576: }
  577:
  578:
  579: // ---- EX12 ----
  580: void ParMesh::Average(std::vector<double> &w) const
  581: {
  582:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  583:     {
  584:         _sendbuf[ls] = w[_buf2loc.at(ls)];
  585:     }
  586:     int ierr = MPI_Alltoallv(MPI_IN_PLACE, _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE,
  587:                              _sendbuf.data(), _sendcounts.data(), _sdispls.data(), MPI_
DOUBLE, _icomm);
  588:     assert(ierr==0);
  589:     //DebugVector(_sendbuf,"_sendbuf",_icomm);
  590:
  591:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) w[_loc_itf.at(lk)] = 0.0;   // only
 for interface nodes
  592:     for(size_t ls = 0; ls<_sendbuf.size(); ++ls)
  593:     {
  594:         w[_buf2loc.at(ls)] += _sendbuf[ls];
  595:     }
  596:
  597:     // Divide interface nodes value by its valence
  598:     for(size_t lk = 0; lk<_loc_itf.size(); ++lk) w[_loc_itf.at(lk)] /= _valence[_loc
_itf.at(lk)];
  599: }
  600:
  601:
  602:
  603:
  604:
  605:
  606:
  607:
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  609:
```

```
  610:
  611:
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```

```
    1: #ifndef PAR_GEOM_FILE
    2: #define PAR_GEOM_FILE
    3: #include "geom.h"
    4: #include "vdop.h"
    5: #include <array>
    6: #include <functional>              // function; C++11
    7: #include <iostream>
    8: #include <map>
    9: #include <memory>                  // shared_ptr
   10: #include <mpi.h>                   // MPI
   11: #include <string>
   12: #include <vector>
   13:
   14: class ParMesh: public Mesh
   15: {
   16: public:
   17:     /**
   18:      * Constructor initializing the members with default values.
   19:      *
   20:      * @param[in] ndim     space dimensions (dimension for coordinates)
   21:      * @param[in] nvert_e  number of vertices per element (dimension for connectivi
ty)
   22:      * @param[in] ndof_e   degrees of freedom per element (= @p nvert_e for linear
elements)
   23:      * @param[in] nedge_e  number of edges per element (= @p nvert_e for linear ele
ments in 2D)
   24:      * @param[in] icomm    MPI communicator
   25:      */
   26:     explicit ParMesh(int ndim, int nvert_e = 0, int ndof_e = 0, int nedge_e = 0, MPI
_Comm const &icomm = MPI_COMM_WORLD);
   27:
   28:     ParMesh(ParMesh const &) = default;
   29:
   30:     ParMesh &operator=(ParMesh const &) = delete;
   31:
   32:
   33:     /**
   34:      * Destructor.
   35:      *
   36:      * See clang warning on
   37:      * <a href="https://stackoverflow.com/questions/28786473/clang-no-out-of-line-vi
rtual-method-definitions-pure-abstract-c-class/40550578">weak-vtables</a>.
   38:      */
   39:     virtual ~ParMesh();
   40:
   41:     /**
   42:      * Reads mesh data from a binary file.
   43:      *
   44:      * @param[in] sname suffix of file name
   45:      * @param[in] icomm  MPI communicator
   46:      * @see  ascii_write_mesh.m for the file format.
   47:     */
   48:     explicit ParMesh(std::string const &sname, MPI_Comm const &icomm = MPI_COMM_WORL
D);
   49:
   50:     void VecAccu(std::vector<double> &w) const;
   51:     void VecAccuInt(std::vector<int> &w) const;
   52:     int GlobalNodes() const;
   53:     void Average(std::vector<double> &w) const;
   54:
   55:     /**  Inner product
   56:      * @param[in] x  vector
   57:      * @param[in] y  vector
   58:      * @return       resulting Euclidian inner product <x,y>
   59:     */
   60:     double dscapr(std::vector<double> const &x, std::vector<double> const &y) const
   61:     {
   62:         return par_scalar(x, y, _icomm);
```

```
    63:     }
    64:
    65: private:
    66:     /**
    67:      * Reads the global triangle to subdomain mapping.
    68:      *
    69:      * @param[in] dname file name
    70:      *
    71:      * @see ascii_write_subdomains.m for the file format
    72:     */
    73:     std::vector<int> ReadElementSubdomains(std::string const &dname);
    74:
    75:
    76:     /**
    77:      * Transform
    78:       *
    79:      * @param[in] myrank MPI rank of this process
    80:       * @param[in] t2d       global mapping triangle to subdomain for all elements (
vertex based)
    81:     */
    82:     void Transform_Local2Global_Vertex(int myrank, std::vector<int> const &t2d);
    83:
    84:
    85:     /**
    86:      * Transform
    87:     */
    88:     void Generate_VectorAdd();
    89:
    90:     bool CheckInterfaceExchange_InPlace() const;
    91:     bool CheckInterfaceExchange() const;
    92:     bool CheckInterfaceAdd_InPlace() const;
    93:     bool CheckInterfaceAdd() const;
    94:
    95:
    96: public:
    97:     /**     MPI rank of the calling process in communication group.
    98:     *
    99:     * @return       MPI rank of the calling process
   100:     */
   101:     int MyRank() const
   102:     {
   103:         return _myrank;
   104:     }
   105:
   106:     /**  Number of MPI processes in communication group.
   107:      *
   108:      * @return       Number of MPI processes
   109:     */
   110:     int NumProcs() const
   111:     {
   112:         return _numprocs;
   113:     }
   114:
   115:     /**  Returns recent
   116:      * @return       MPI communicator
   117:     */
   118:     MPI_Comm GetCommunicator() const
   119:     {
   120:         return _icomm;
   121:     }
   122:
   123: private:
   124:     // Don't use  &_icomm  ==> Error
   125:     MPI_Comm const _icomm;                      //!< MPI communicator for the group o
f processes
   126:     int _numprocs;                              //!< number of MPI processes
   127:     int _myrank;                                //!< my MPI rank
   128:     std::vector<int> _v_l2g;                    //!< vertices:  local to global mappi
```

```
ng
  129:     std::vector<int> _t_l2g;                  //!< triangles: local to global mappi
ng
  130:     std::map<int, int> _v_g2l;                //!< vertices:  global to local mappi
ng
  131:     std::map<int, int> _t_g2l;                //!< triangles: global to local mappi
ng
  132:
  133:     //std::vector<int> e_l2g;                   //!< edges:     local to global map
ping
  134:
  135:     std::vector<int> _valence;                //!< valence of local vertices, i.e.
number of subdomains they belong to
  136:     //  MPI_Alltoallv needs:
  137:     mutable std::vector<double> _sendbuf;     //!< send buffer  a n d  receiving bu
ffer (MPI_IN_PLACE)
  138:     std::vector<int>    _sendcounts;          //!< number of data to send to each M
PI rank (the same as for recv)
  139:     std::vector<int>    _sdispls;             //!< offset of data to send to each M
PI rank wrt. _senbuffer (the same as for recv)
  140:     //
  141:     //  We need to map the interface vertices onto the sendbuffer:
  142:     std::vector<int> _loc_itf;                //!< local  index of interface vertex
 lk
  143:     std::vector<int> _gloc_itf;               //!< global index of interface vertex
 lk
  144:     std::vector<int> _buf2loc;                //!< local indices of sendbuffer posi
tions (the same as for recv)
  145:
  146:
  147: };
  148:
  149:
  150: #endif
```

```cpp
#include "vdop.h"
#include <cassert>                // assert()
#include <cmath>
#include <iostream>
#include <vector>
using namespace std;


void vddiv(vector<double> & x, vector<double> const& y,
                               vector<double> const& z)
{
    assert( x.size()==y.size() && y.size()==z.size() );
    size_t n = x.size();

#pragma omp parallel for
    for (size_t k = 0; k < n; ++k)
    {
        x[k] = y[k] / z[k];
    }
    return;
}

//****************************************************************************

void vdaxpy(std::vector<double> & x, std::vector<double> const& y,
                        double alpha, std::vector<double> const& z )
{
    assert( x.size()==y.size() && y.size()==z.size() );
    size_t n = x.size();

#pragma omp parallel for
    for (size_t k = 0; k < n; ++k)
    {
        x[k] = y[k] + alpha * z[k];
    }
    return;
}
//****************************************************************************

double dscapr(std::vector<double> const& x, std::vector<double> const& y)
{
    assert( x.size()==y.size());
    size_t n = x.size();

    double    s = 0.0;
//#pragma omp parallel for reduction(+:s)
    for (size_t k = 0; k < n; ++k)
    {
        s += x[k] * y[k];
    }

    return s;
}

//****************************************************************************
//void DebugVector(vector<double> const &v)
//{
    //cout << "\nVector  (nnode = " << v.size() << ")\n";
    //for (size_t j = 0; j < v.size(); ++j)
    //{
        //cout.setf(ios::right, ios::adjustfield);
        //cout << v[j] << "   ";
    //}
    //cout << endl;

    //return;
//}
//****************************************************************************
```

```
   69: bool CompareVectors(std::vector<double> const& x, int const n, double const y[], dou
ble const eps)
   70: {
   71:     bool bn = (static_cast<int>(x.size())==n);
   72:     if (!bn)
   73:     {
   74:         cout << "#########   Error: " << "number of elements" << endl;
   75:     }
   76:     //bool bv = equal(x.cbegin(),x.cend(),y);
   77:     bool bv = equal(x.cbegin(),x.cend(),y,
   78:                     [eps](double a, double b) -> bool
   79:                     { return std::abs(a-b)<eps*(1.0+0.5*(std::abs(a)+ std::abs
(b))); }
   80:     );
   81:     if (!bv)
   82:     {
   83:         assert(static_cast<int>(x.size())==n);
   84:         cout << "#########   Error: " << "values" << endl;
   85:     }
   86:     return bn && bv;
   87: }
   88:
   89: //*************************************************************************
   90: double par_scalar(vector<double> const &x, vector<double> const &y, MPI_Comm const&
icomm)
   91: {
   92:   const double s = dscapr(x,y);
   93:         double sg;
   94:   MPI_Allreduce(&s,&sg,1,MPI_DOUBLE,MPI_SUM,icomm);
   95:
   96:   return(sg);
   97: }
   98:
   99: //*************************************************************************
  100: void ExchangeAll(vector<double> const &xin, vector<double> &yout, MPI_Comm const &ic
omm)
  101: {
  102:     int myrank, numprocs,ierr(-1);
  103:     MPI_Comm_rank(icomm, &myrank);                          // my MPI-rank
  104:     MPI_Comm_size(icomm, &numprocs);
  105:     int const N=xin.size();
  106:     int const sendcount = N/numprocs;                       // equal sized junks
  107:     assert(sendcount*numprocs==N);                          // really all junk sized
?
  108:     assert(xin.size()==yout.size());
  109:
  110:     auto sendbuf = xin.data();
  111:     auto recvbuf = yout.data();
  112:     ierr = MPI_Alltoall(sendbuf, sendcount, MPI_DOUBLE,
  113:                         recvbuf, sendcount, MPI_DOUBLE, icomm);
  114:     assert(0==ierr);
  115:
  116:     return;
  117: }
  118:
  119: //*************************************************************************
  120: void ExchangeAllInPlace(vector<double> &xin, MPI_Comm const &icomm)
  121: {
  122:     int myrank, numprocs,ierr(-1);
  123:     MPI_Comm_rank(icomm, &myrank);                          // my MPI-rank
  124:     MPI_Comm_size(icomm, &numprocs);
  125:     int const N=xin.size();
  126:     int const sendcount = N/numprocs;                       // equal sized junks
  127:     assert(sendcount*numprocs==N);                          // really all junk sized
?
  128:
  129:     auto sendbuf = xin.data();
  130:     ierr = MPI_Alltoall(MPI_IN_PLACE, sendcount, MPI_DOUBLE,
```

```
  131:                          sendbuf, sendcount, MPI_DOUBLE, icomm);
  132:     assert(0==ierr);
  133:
  134:     return;
  135: }
```

```
    1: #ifndef VDOP_FILE
    2: #define VDOP_FILE
    3: #include <iostream>
    4: #include <mpi.h>                    // MPI
    5: #include <string>
    6: #include <vector>
    7:
    8: /** @brief  Element-wise vector divison x_k = y_k/z_k.
    9:  *
   10:  * @param[out] x  target vector
   11:  * @param[in]  y  source vector
   12:  * @param[in]  z  source vector
   13:  *
   14:  */
   15: void vddiv(std::vector<double> &x, std::vector<double> const &y,
   16:            std::vector<double> const &z);
   17:
   18: /** @brief  Element-wise daxpy operation x(k) = y(k) + alpha*z(k).
   19:  *
   20:  * @param[out] x  target vector
   21:  * @param[in]  y  source vector
   22:  * @param[in]  alpha  scalar
   23:  * @param[in]  z  source vector
   24:  *
   25:  */
   26: void vdaxpy(std::vector<double> &x, std::vector<double> const &y,
   27:             double alpha, std::vector<double> const &z );
   28:
   29:
   30: /** @brief  Calculates the Euclidean inner product of two vectors.
   31:  *
   32:  * @param[in]  x vector
   33:  * @param[in]  y vector
   34:  * @return Euclidean inner product @f$\langle x,y \rangle@f$
   35:  *
   36:  */
   37: double dscapr(std::vector<double> const &x, std::vector<double> const &y);
   38:
   39:
   40: inline
   41: double L2_scapr(std::vector<double> const &x, std::vector<double> const &y)
   42: {
   43:     return dscapr(x, y) / x.size();
   44: }
   45:
   46:
   47: /**     Parallel inner product
   48:   @param[in] x      vector
   49:   @param[in] y      vector
   50:   @param[in] icomm  MPI communicator
   51:   @return               resulting Euclidian inner product <x,y>
   52: */
   53: double par_scalar(std::vector<double> const &x, std::vector<double> const &y,
   54:                   MPI_Comm const& icomm=MPI_COMM_WORLD);
   55:
   56:
   57:
   58: /*  ReadId : Input and broadcast of an integer */
   59: inline
   60: int ReadIn(std::string const &ss = std::string(), MPI_Comm const &icomm = MPI_COMM_W
ORLD)
   61: {
   62:     MPI_Barrier(icomm);
   63:     int myrank;                            /*  my rank number */
   64:     MPI_Comm_rank(icomm, &myrank);
   65:     int id;
   66:
   67:     if (myrank == 0) {
```

```
   68:         std::cout << "\n\n  " << ss << " :  Which process do you want to debug ? \n"
;
   69:         std::cin >> id;
   70:     }
   71:     MPI_Bcast(&id, 1, MPI_INT, 0, icomm);
   72:
   73:     return id;
   74: }
   75:
   76: /**
   77:  * Print entries of a vector to standard output.
   78:  *
   79:  * @param[in]   v       vector values
   80:  * @param[in]   ss      string containing the vector name
   81:  * @param[in]   icomm   communicator group for MPI
   82:  *
   83: */
   84: //void DebugVector(std::vector<double> const &v);
   85: template <class T>
   86: void DebugVector(std::vector<T> const &v, std::string const &ss = std::string(), MPI
_Comm const &icomm = MPI_COMM_WORLD)
   87: {
   88:     MPI_Barrier(icomm);
   89:     int numprocs;                                   /*  # processes    */
   90:     MPI_Comm_size(icomm, &numprocs);
   91:     int myrank;                                 /*  my rank number */
   92:     MPI_Comm_rank(icomm, &myrank);
   93:
   94:     int readid = ReadIn(ss);                            /* Read  readid    */
   95:
   96:     while ( (0 <= readid) && (readid < numprocs) ) {
   97:         if (myrank == readid) {
   98:             std::cout << "\n\n process " << readid;
   99:             std::cout << "\n ....  " << ss << " (nnode = " << v.size() << ")\n";
  100:             for (size_t j = 0; j < v.size(); ++j) {
  101:                 std::cout.setf(std::ios::right, std::ios::adjustfield);
  102:                 std::cout << v[j] << "   ";
  103:             }
  104:             std::cout << std::endl;
  105:             fflush(stdout);
  106:         }
  107:
  108:         readid = ReadIn(ss, icomm);                     /* Read  readid    */
  109:     }
  110:     MPI_Barrier(icomm);
  111:     return;
  112: }
  113:
  114: /** @brief  Compares an STL vector with POD vector.
  115:  *
  116:  * The accuracy criteria @f$ |x_k-y_k| < \varepsilon \left({1+0.5(|x_k|+|y_k|)}\righ
t) @f$
  117:  * follows the book by
  118:  * <a href="https://www.springer.com/la/book/9783319446592">Stoyan/Baran</a>, p.8.
  119:  *
  120:  * @param[in]  x    STL vector
  121:  * @param[in]  n    length of POD vector
  122:  * @param[in]  y    POD vector
  123:  * @param[in]  eps  relative accuracy criteria (default := 0.0).
  124:  * @return true iff pairwise vector elements are relatively close to each other.
  125:  *
  126:  */
  127: bool CompareVectors(std::vector<double> const &x, int n, double const y[], double co
nst eps = 0.0);
  128:
  129:
  130: /** Output operator for vector
  131:  *      @param[in,out] s        output stream, e.g. @p cout
```

```
  132:  *  @param[in]     v    vector
  133:  *
  134:  *      @return    output stream
  135: */
  136: template <class T>
  137: std::ostream &operator<<(std::ostream &s, std::vector<T> const &v)
  138: {
  139:     for (auto vp : v) {
  140:         s << vp << "  ";
  141:     }
  142:     return s;
  143: }
  144:
  145: /** Exchanges equal size partions of vector @p xin with all MPI processes.
  146:  * The received data are return in vector @p yout .
  147:  *
  148:  *  @param[in]  xin   input vector
  149:  *  @param[out] yout  output vector
  150:  *  @param[in]  icomm MPI communicator
  151:  *
  152: */
  153: void ExchangeAll(std::vector<double> const &xin, std::vector<double> &yout, MPI_Comm
 const &icomm = MPI_COMM_WORLD);
  154:
  155: /** Exchanges equal size partions of vector @p xin with all MPI processes.
  156:  * The received data are return in vector @p xin  .
  157:  *
  158:  *  @param[in,out]  xin   input/output vector
  159:  *  @param[in]  icomm MPI communicator
  160:  *
  161: */
  162: void ExchangeAllInPlace(std::vector<double> &xin, MPI_Comm const &icomm = MPI_COMM_W
ORLD);
  163:
  164:
  165:
  166: #endif
```