All-distances-twice rows

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What is a tone-row?

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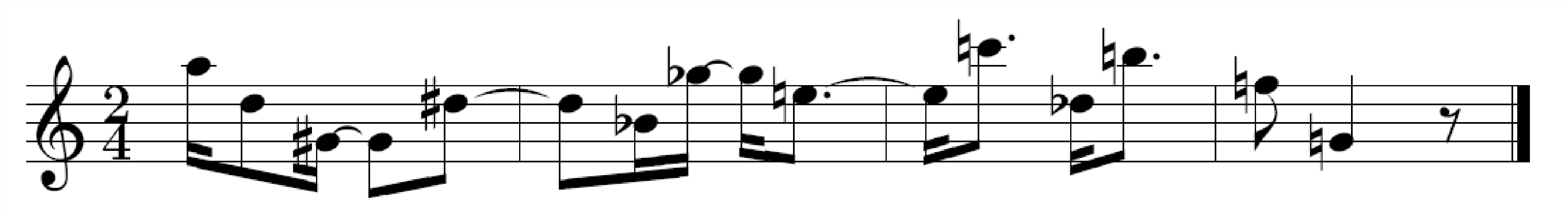

O. Messiaen: Le Merle Noir: A piece for flute and piano composed in 1951. The tone row appears in the coda of the piano part.

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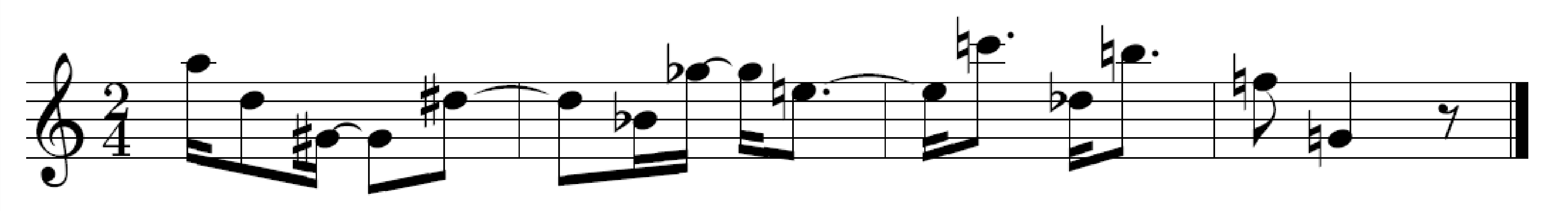

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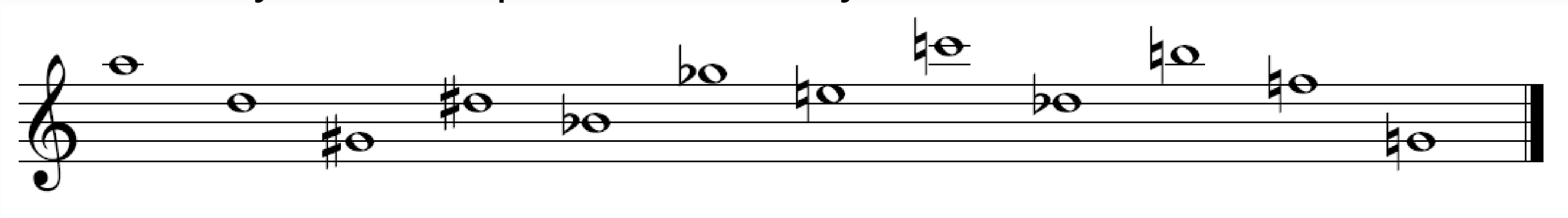

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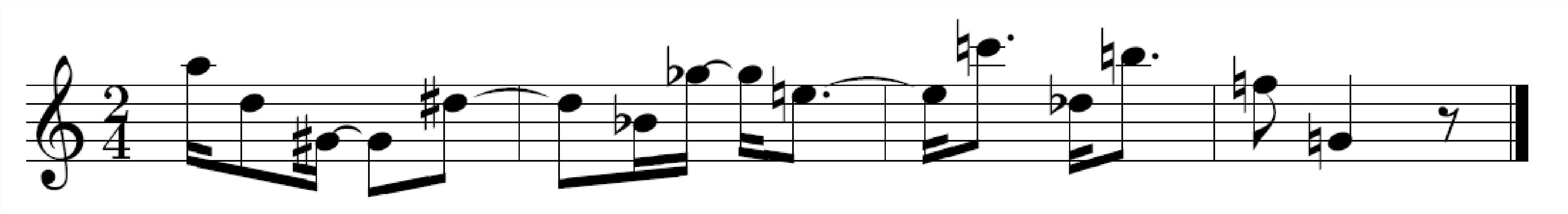

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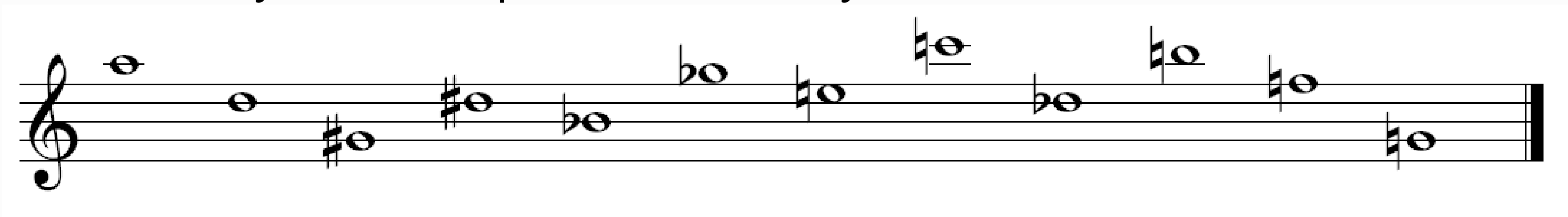
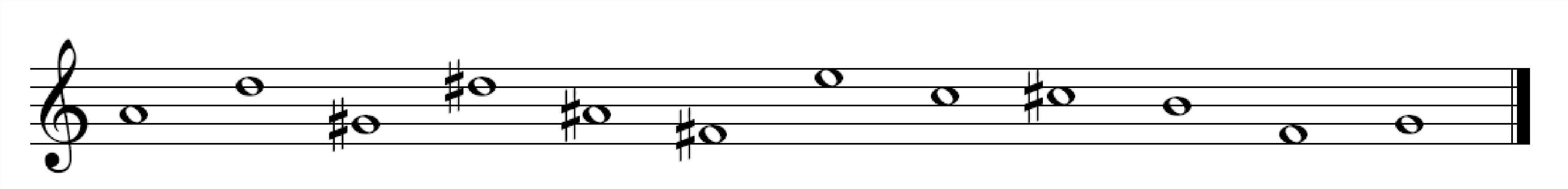
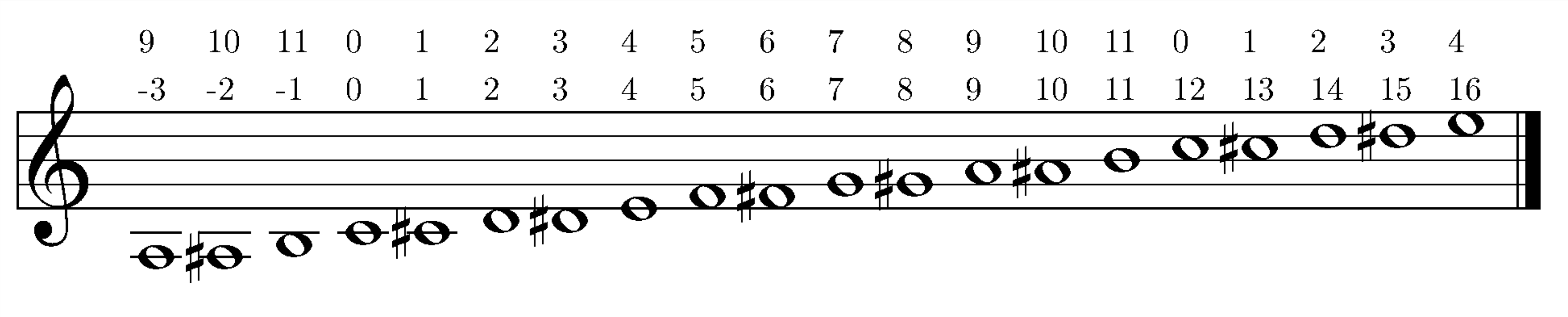


Replacing the tones by representatives of octave equivalence



each of the 12 possible octave equivalence classes occurs exactly once.

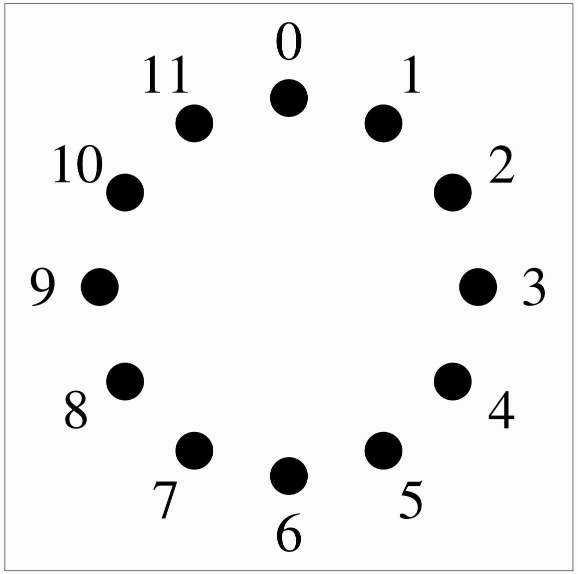
The chromatic scale



Here is a short part of the chromatic scale together with the labelling of the tones in $\mathbb{Z}$ and pitch classes in $\mathbb{Z}_{12}$.

Pitch classes are the equivalence classes with respect to octave equivalence. They form the cyclic group $(\mathbb{Z}_{12}, +)$ or the residue class ring $(\mathbb{Z}_{12}, +\cdot)$, where $\mathbb{Z}_{12} := \mathbb{Z} \bmod 12\mathbb{Z} = \{\bar{0}, \bar{1}, \ldots, \overline{11}\}$.

Circular representation of $\mathbb{Z}_{12}$:



Tone rows

In Music: a sequence of 12 tones so that different tones are not octave equivalent.

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In Mathematics:

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Simple model:

$$f\colon\{1,\dots,12\}\to\mathbb{Z}_{12}, \quad \text{bijective.}$$

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Simple model:

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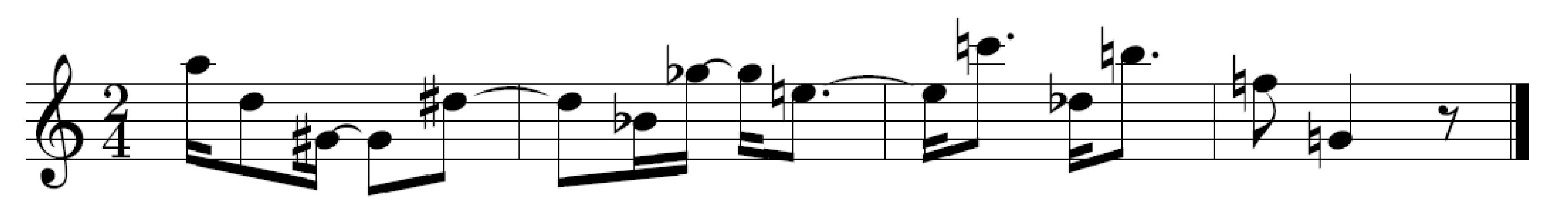
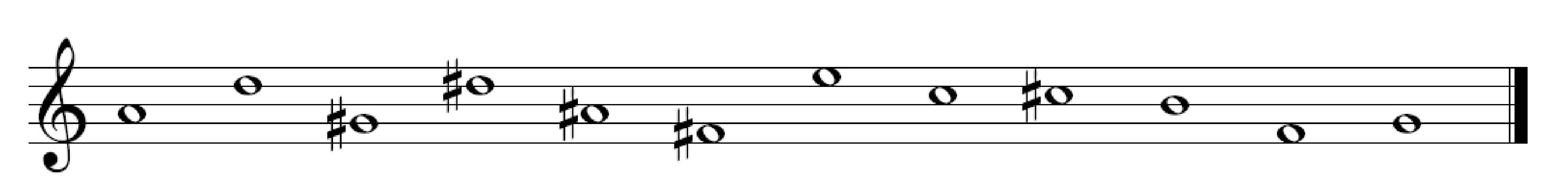
Each pitch class occurs ***exactly*** once as $f(j)$, $j \in \{1,\dots,12\}$.

O. Messiaen: Le Merle Noir



Reduction to pitch classes $(9, 2, 8, 3, 10, 6, 4, 0, 1, 11, 5, 7)$.



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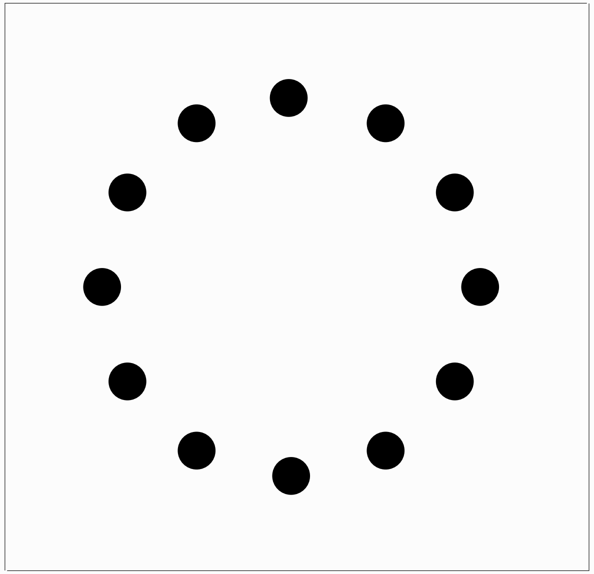

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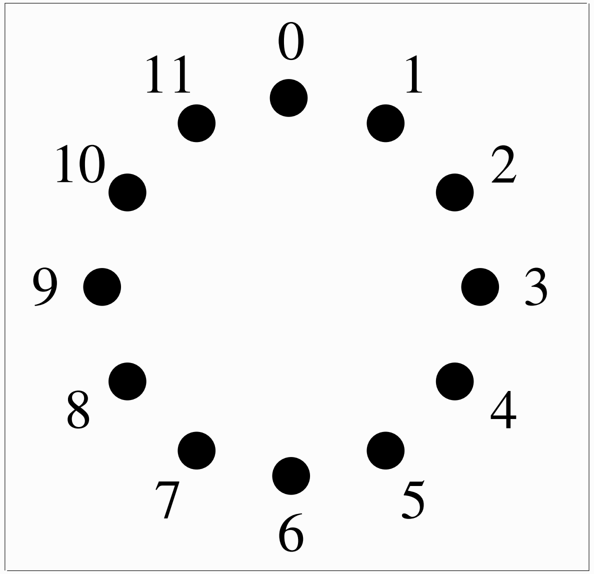

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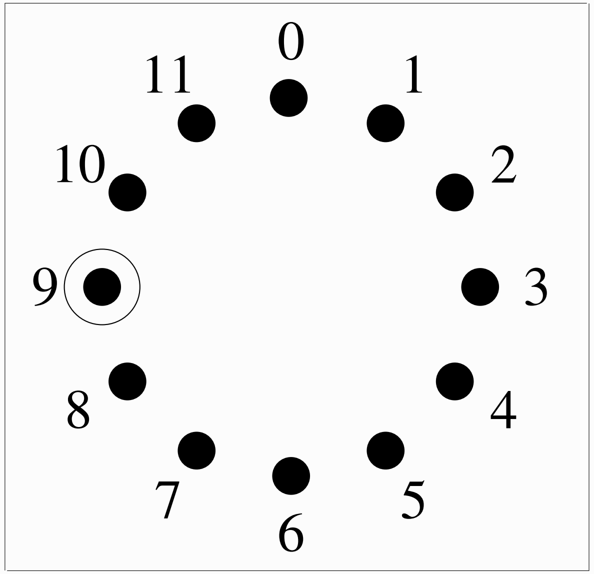

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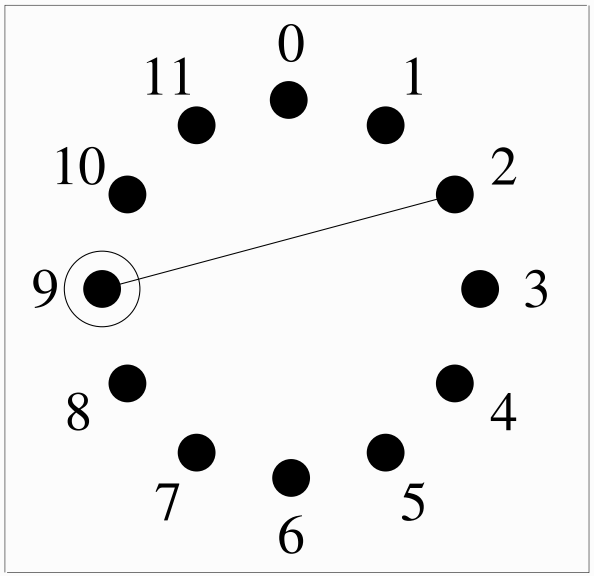
and we connect pitch classes which occur in consecutive positions in the tone row.



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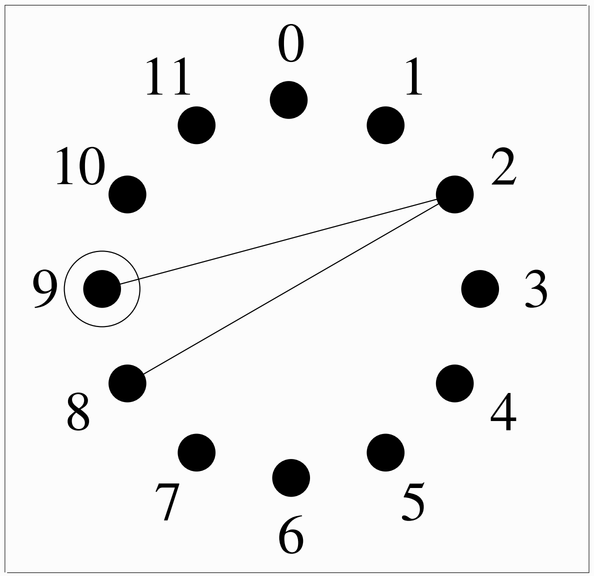

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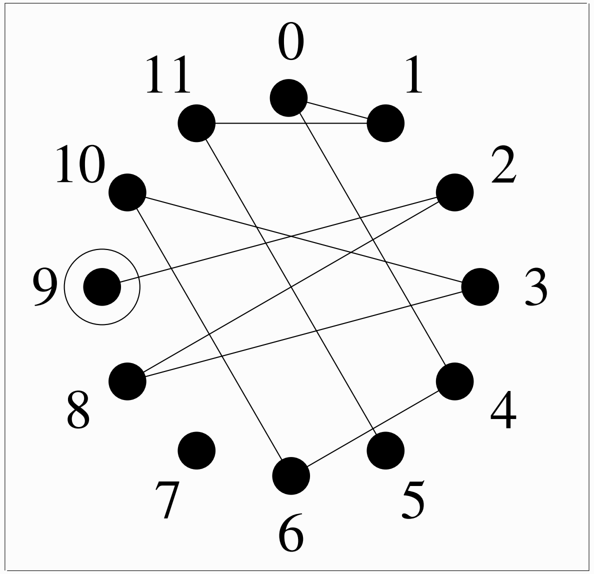

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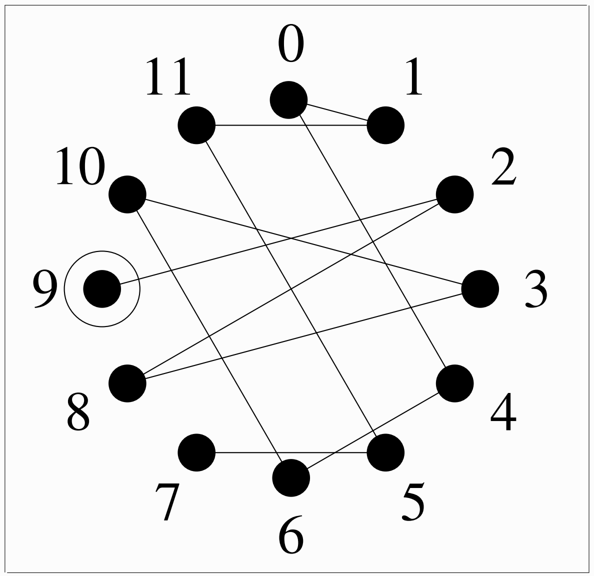

Intervals

The interval from pitch class a to pitch class b is the difference

$$b - a \in \mathbb{Z}_{12}.$$

It is the number of steps in clockwise direction from a to b.

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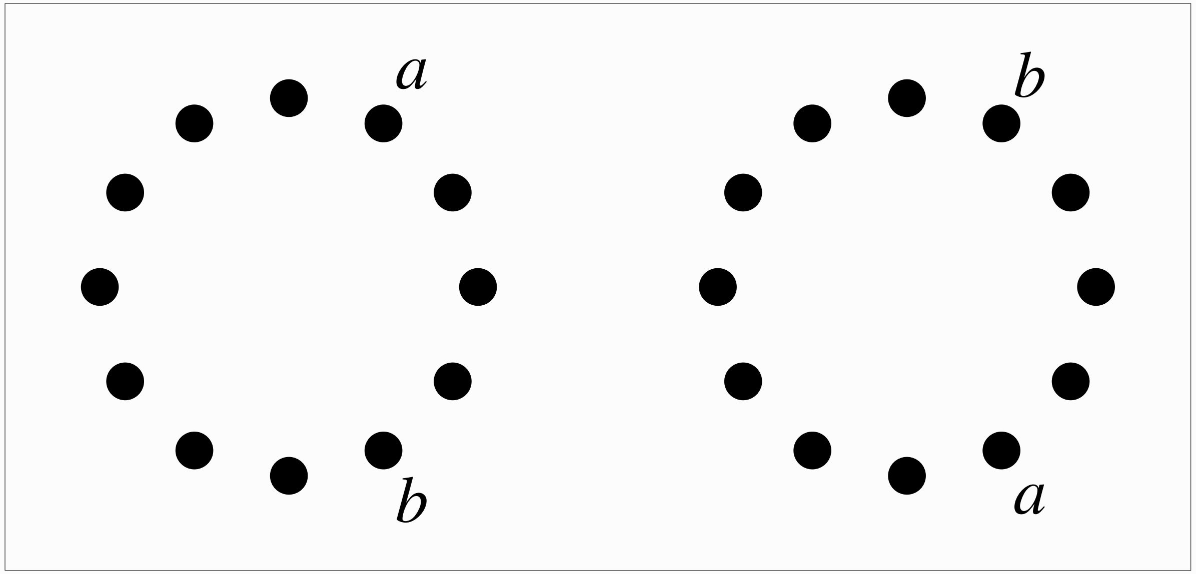

The interval from a to b is 4, respectively 8.

Interval Vector

Given a tone row f, the interval vector is the sequence of eleven intervals

$$g = (f(2) - f(1), f(3) - f(2), \ldots, f(12) - f(11)).$$

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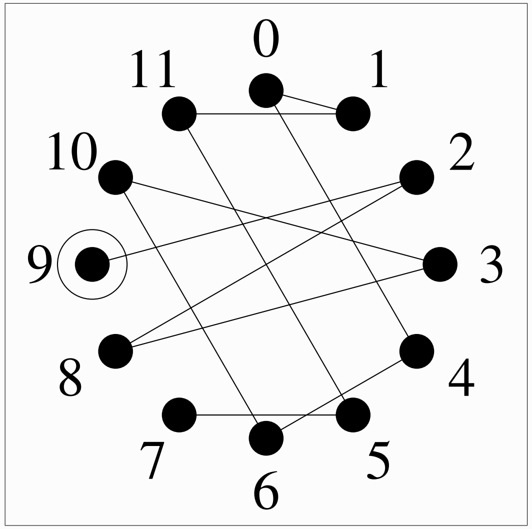
The interval vector of $f = (9,2,8,3,10,6,4,0,1,11,5,7)$



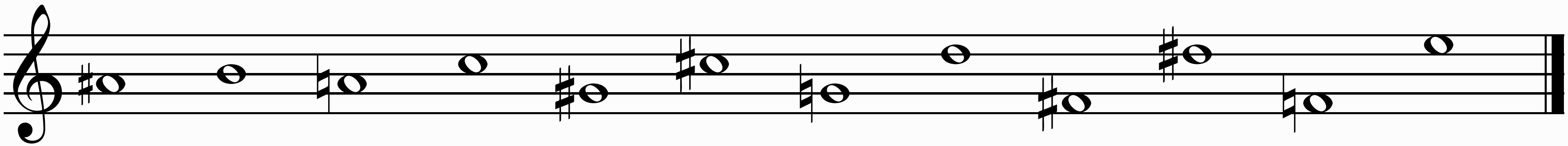
is $(5,6,7,7,8,10,8,1,10,6,2)$.

All-interval row

A tone row is an all interval row if each interval occurs exactly once in its interval vector.

Obviously the previous tone row is not an all-interval row.

Consider $f = (10, 11, 9, 0, 8, 1, 7, 2, 6, 3, 5, 4)$



Then the interval vector is $(1, 10, 3, 8, 5, 6, 7, 4, 9, 2, 11)$.

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Equivalence classes of tone rows

There are

$$12! = 12 \cdot 11 \cdots 2 \cdot 1 = 479\,001\,600$$

different tone rows.

We are looking for a suitable equivalence relation:
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Reduction of the number of tone rows to the number of different similarity classes of tone rows.

Equivalence by Schönberg: Tone rows are equivalent whenever they can be constructed by transposing, inversion and/or retrograde from a single tone row.

Transposing

of pitch classes:

$$T\colon \mathbb{Z}_{12} \to \mathbb{Z}_{12}, \qquad i \mapsto i+1,$$

or for $r \in \mathbb{Z}$

$$T^r\colon \mathbb{Z}_{12} \to \mathbb{Z}_{12}, \qquad i \mapsto i+\bar{r}.$$

of tone rows:

$$f \mapsto T^r \circ f.$$

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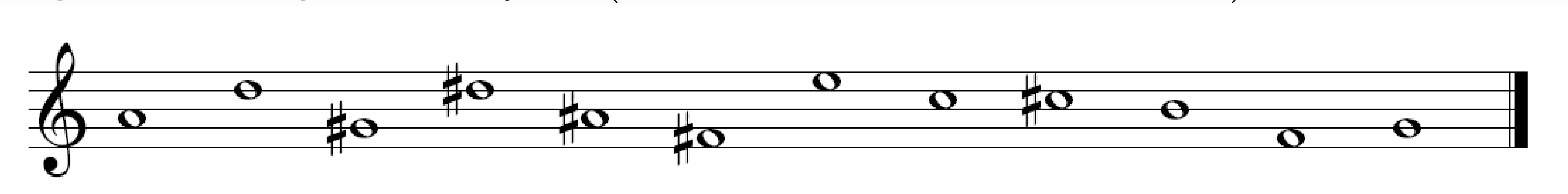
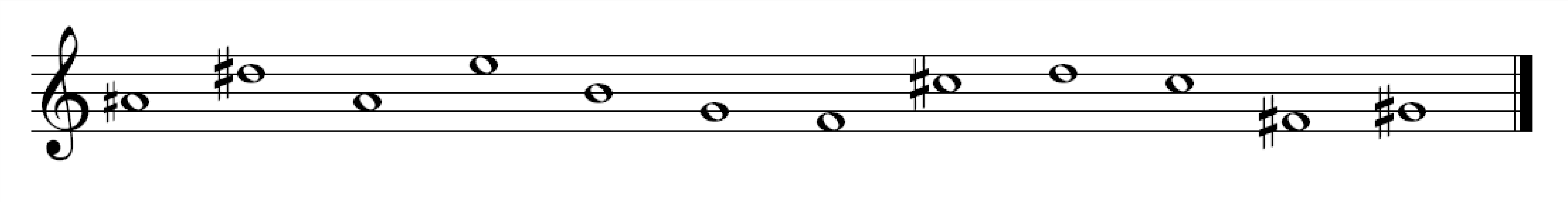
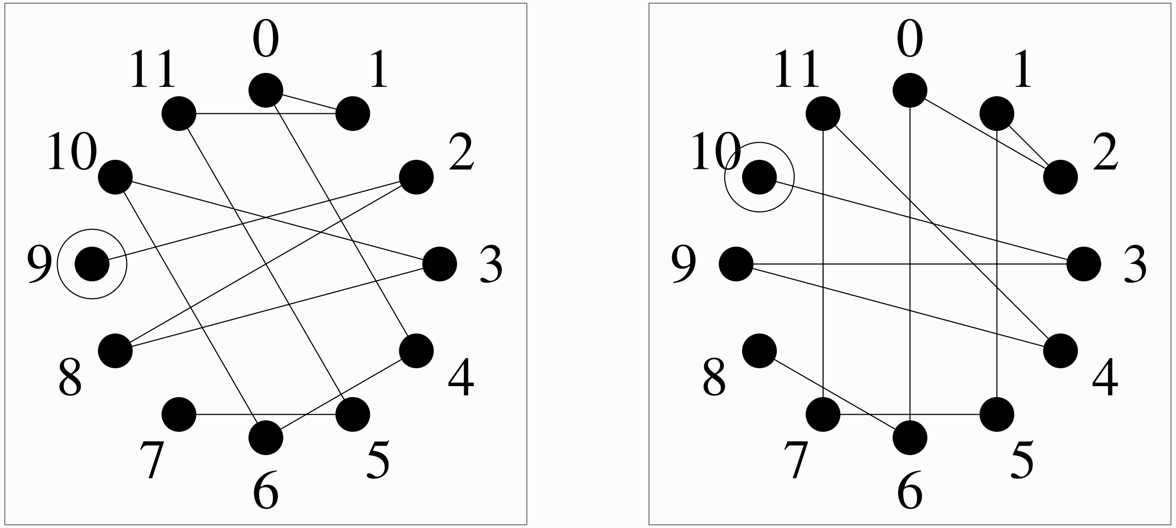
E.g. the transposed of $f = (9,2,8,3,10,6,4,0,1,11,5,7)$



is $T \circ f = (10,3,9,4,11,7,5,1,2,0,6,8)$



Rotation of the circular representation.



The interval vector of $T^r \circ f$ coincides with the interval vector of f. Actually, all tone rows having the same interval vector as f are transpositions of f.

If f is an all-interval row, then so is $T^r \circ f$.

Inversion

of pitch classes:

$$I\colon \mathbb{Z}_{12} \to \mathbb{Z}_{12}, \qquad i \mapsto -i,$$

or for $r \in \mathbb{Z}$

$$T^r \circ I\colon \mathbb{Z}_{12} \to \mathbb{Z}_{12}, \qquad i \mapsto -i + \bar{r}.$$

of tone rows:

$$f \mapsto (T^r \circ I) \circ f.$$

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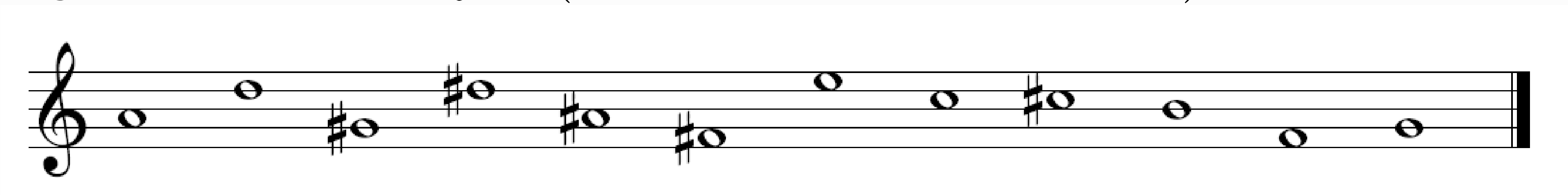
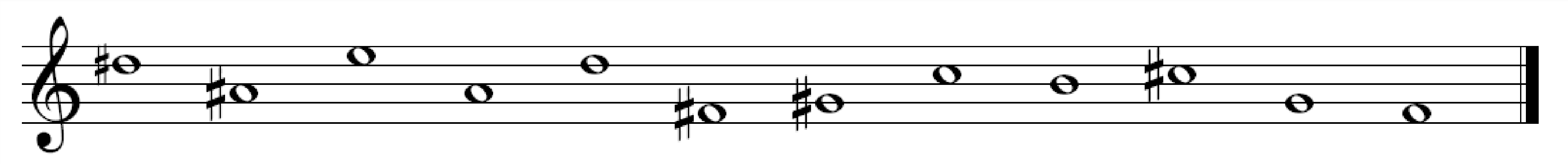
of tone rows:

$$f \mapsto (T^r \circ I) \circ f.$$

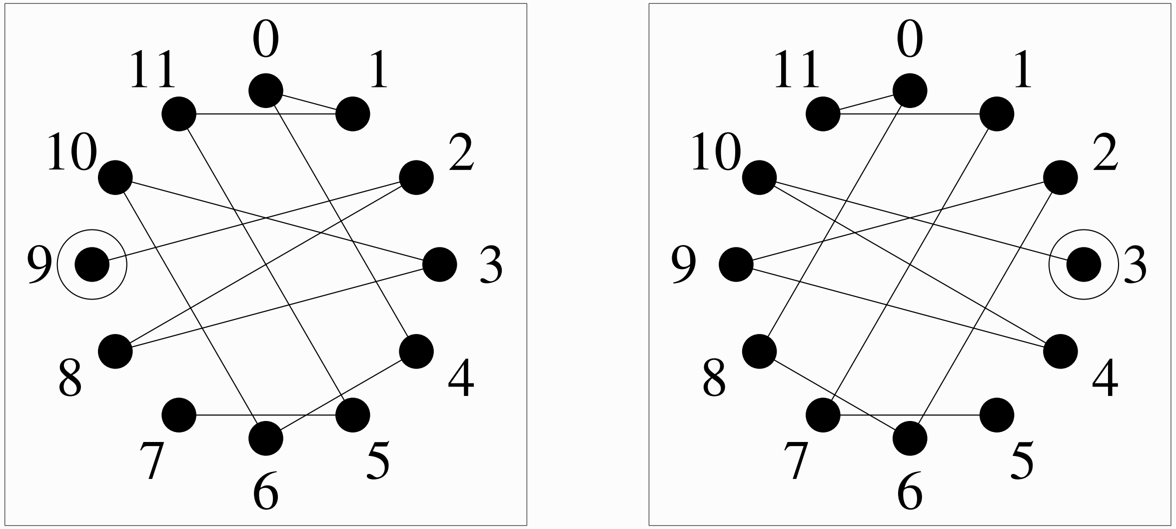
E.g. the inversion of $f = (9, 2, 8, 3, 10, 6, 4, 0, 1, 11, 5, 7)$



is $I \circ f = (3, 10, 4, 9, 2, 6, 8, 0, 11, 1, 7, 5)$



Mirror image of the circular representation.



Let g be the interval vector of f, then the interval vector of $T^r \circ I \circ f$ is $I \circ g$ since

$$\begin{aligned}(T^r \circ I)(f(j+1)) - (T^r \circ I)(f(j)) &= (r - f(j+1)) - (r - f(j)) \\ &= f(j) - f(j+1) \\ &= -(f(j+1) - f(j)).\end{aligned}$$

If f is an all-interval row, then so is $T^r \circ I \circ f$.

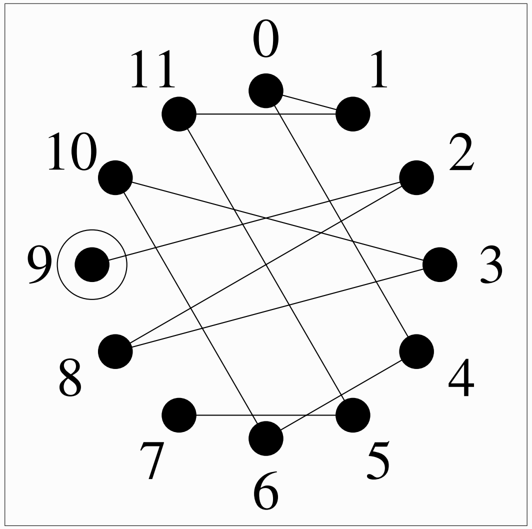
Retrograde of a tone row

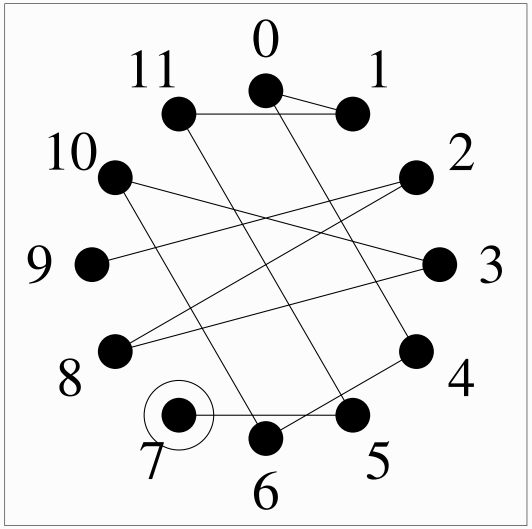
Consider the permutation $R = (1,12)(2,11)\cdots(6,7)$ of $\{1,2,\ldots,12\}$.

Retrograde of f:

$$f \circ R$$

Let $f = (9,2,8,3,10,6,4,0,1,11,5,7)$, then
$f \circ R = (7,5,11,1,0,4,6,10,3,8,2,9)$.





Let g be the interval vector of f, then the interval vector of $f \circ R$ is $I \circ g \circ R$ since

$$\begin{aligned}(f \circ R)(j+1) - (f \circ R)(j) &= f(13-j-1) - f(13-j) \\ &= -(f(13-j) - f((13-j)-1)).\end{aligned}$$

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If f is an all-interval row, then so is $f \circ R$.

Theorem. All tone rows equivalent to an all-interval row in the sense of Schönberg are all-interval rows.

Database of tone rows and tropes

by H.F. and Peter Lackner

In addition to transposition, inversion and retrograde we consider also the ***cyclic shift of a tone row*** f

$$f \circ S, \text{ or } f \circ S^r, \quad r \in \mathbb{Z}$$

as an equivalence relation on tone rows, where $S = (1, 2, \ldots, 12)$.

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Let $f = (9, 2, 8, 3, 10, 6, 4, 0, 1, 11, 5, 7)$, then
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$f \circ S = (2, 8, 3, 10, 6, 4, 0, 1, 11, 5, 7, 9)$.

This reduces the number of 9 985 920 non equivalent tone rows with respect to Schönberg to 836 017 non equivalent tone rows.

One equivalence class contains up to 576 different tone rows.

New equivalence relation for tone rows

f' is ***equivalent*** to f,

- if f' can be constructed from f by any combination of transposing, inversion, cyclic shift and retrograde.

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f' is ***equivalent*** to f,

- if f' can be constructed from f by any combination of transposing, inversion, cyclic shift and retrograde.
- if there exist integers $r, s \in \{0, \ldots, 11\}$ and $v, w \in \{0, 1\}$ so that

$$f' = T^r \circ I^v \circ f \circ R^w \circ S^s.$$

The equivalence classes are the orbits of bijective mappings $f : \mathbb{Z}_{12} \to \mathbb{Z}_{12}$ under the ***group action*** of the direct product of two ***dihedral groups*** of order 24.

This equivalence relations is also used by:

Josef Matthias Hauer (1883-1959) starting with his composition ***Nomos*** op. 19 (1919).

Ronald C. Read. Combinatorial problems in the theory of music. *Discrete Mathematics*, 167–168(1-3):543–551, 1997.

Solomon Wolf Golomb and ***Lloyd Richard Welch***. On the enumeration of polygons. *American Mathematical Monthly*, 87:349–353, 1960.

David J. Hunter and ***Paul T. von Hippel***. How rare is symmetry in musical 12-tone rows? *American Mathematical Monthly*, 110(2):124–132, 2003.

H. F. and ***Peter Lackner***. Tone rows and tropes. *Journal of Mathematics and Music*, 9(2):111–172, 2015.

Circular representation of all equivalent rows:

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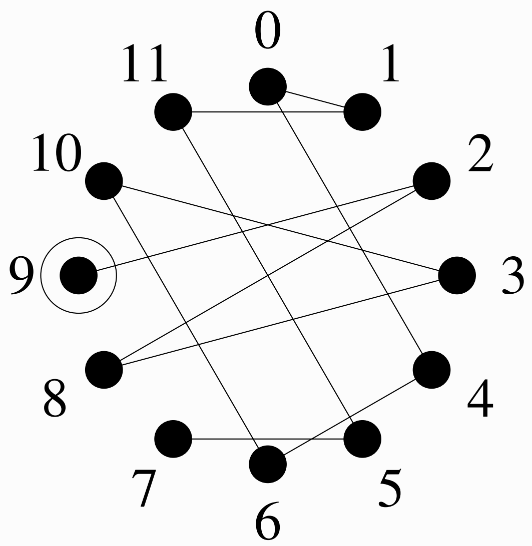
We start with f.



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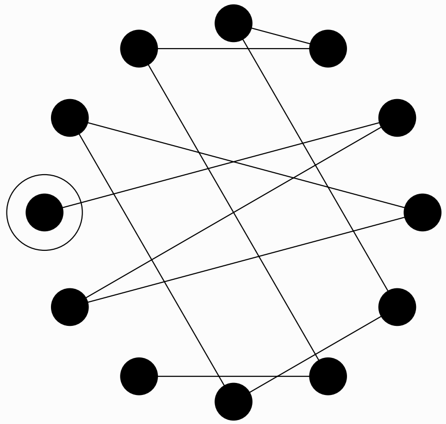
Because of transposing we delete the labels.



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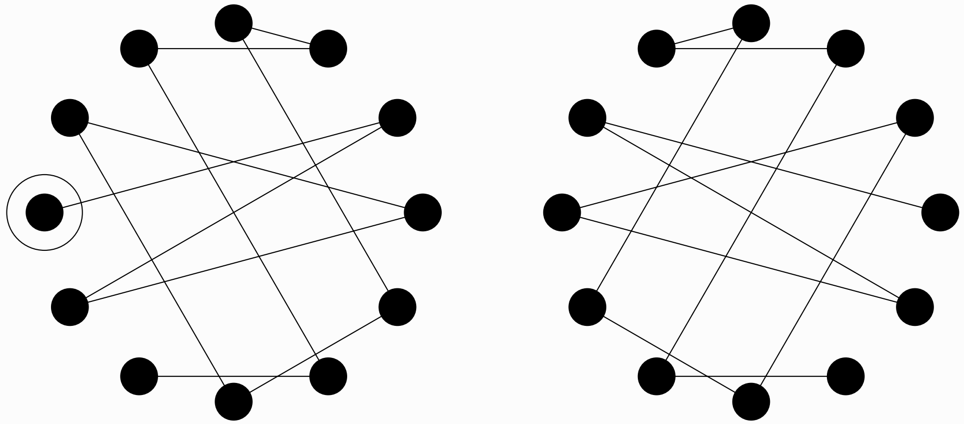
Inversion of f is the mirror of the given graph.



Circular representation of all equivalent rows:

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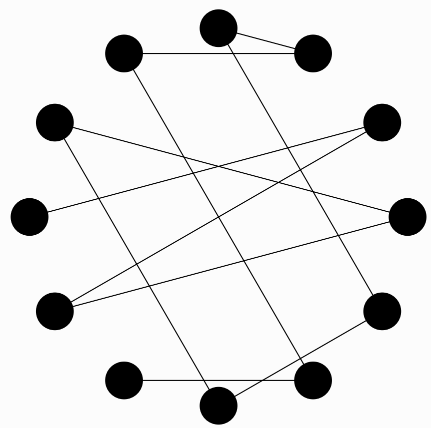
Inversion of f is the mirror of the given graph.

Because of retrograde we don't show the first tone.



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Circular representation of all equivalent rows:

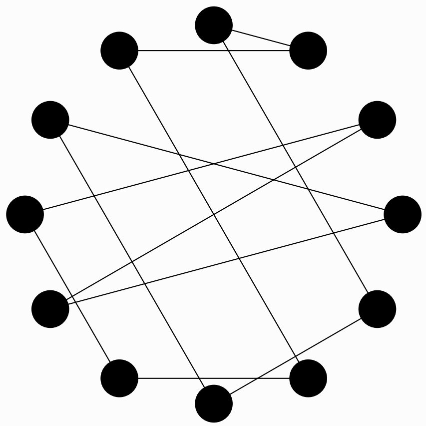
We start with f.

Because of transposing we delete the labels.

Inversion of f is the mirror of the given graph.

Because of retrograde we don't show the first tone.

Because of cyclic shifts we insert the missing edge.



Back to interval vectors and all-interval rows

The interval vector in the new setting consits of 12 intervals, namely

$$g = (f(2)-f(1), f(3)-f(2), \dots, f(12)-f(11), f(1)-f(12))$$

and the interval vector of $f \circ S^r$ is $g \circ S^r$.

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A simple computation shows that if f is an all-interval row, then the last interval $f(1) - f(12) = 6$, therefore the interval vector of an all-interval row contains each interval. The interval 6 occurs twice, all other intervals exactly once.

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Moreover, not all cyclic shifts of an all-interval f row are again all-interval rows. If f is an all-interval row, then there is exactly one cyclic shift $f \circ S^r$, $0 < r < 12$, so that the last member in the interval vector is again equal to 6.

Interval type

The ***interval type*** of a tone row is a vector

$$(a_1, \dots, a_{11})$$

indicating that the interval i occurs exactly a_i times in the interval vector of f.

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The interval types of equivalent tone rows coincide.

An all-interval row has interval type

$$(1,1,1,1,1,2,1,1,1,1,1).$$

Distances

By measuring intervals in both directions we introduce ***distances***.

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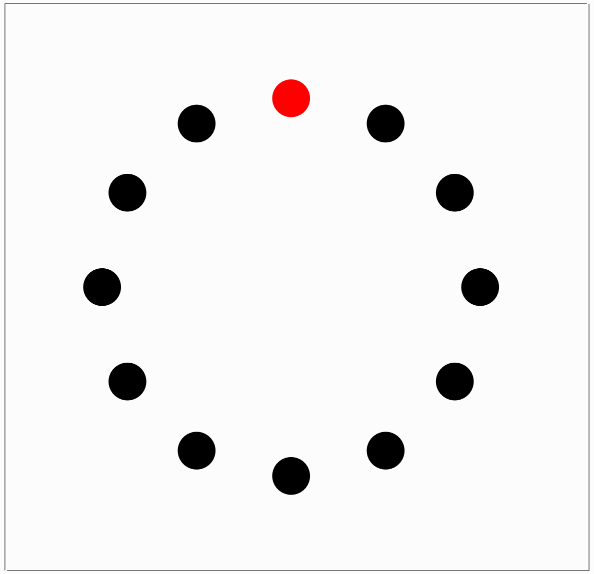

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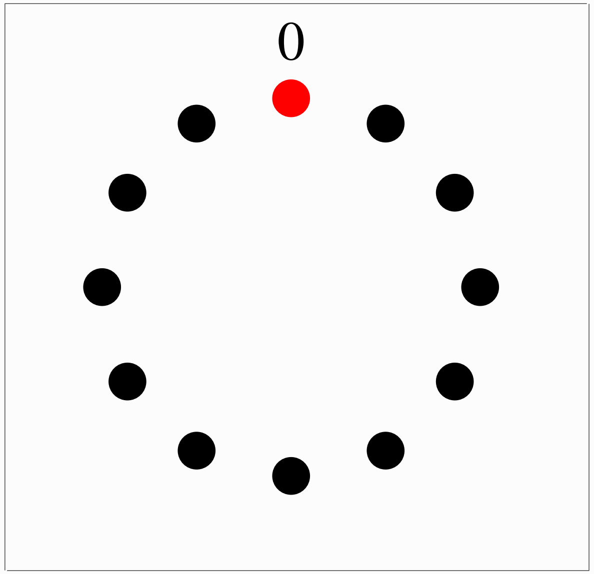

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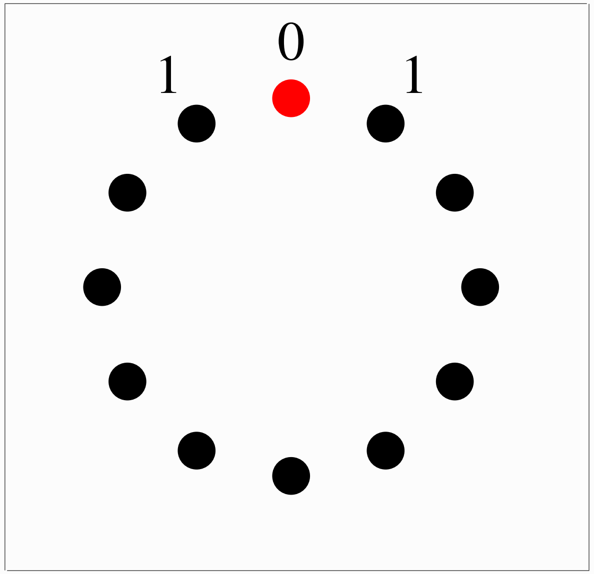

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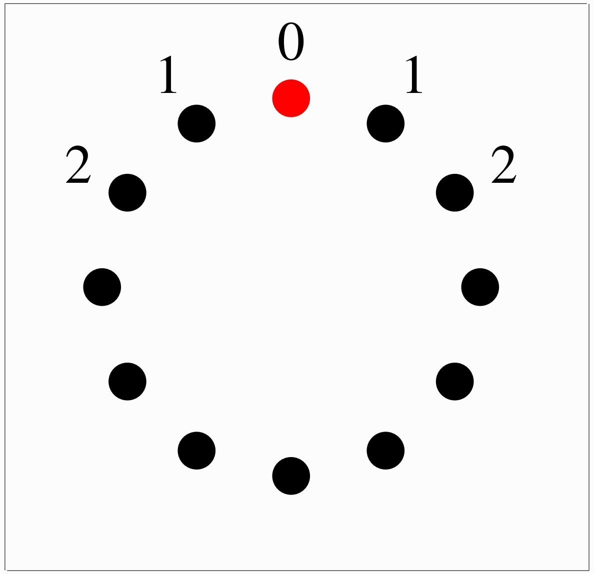

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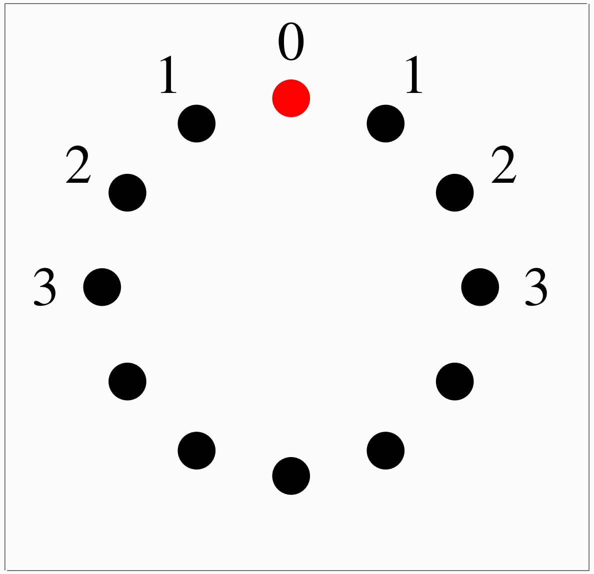

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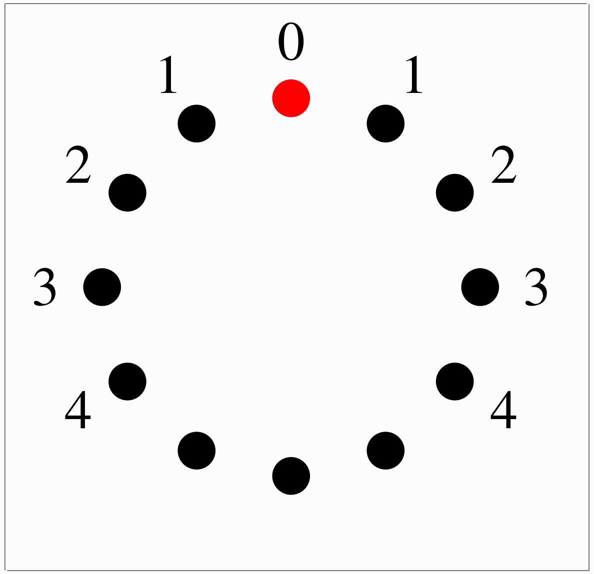

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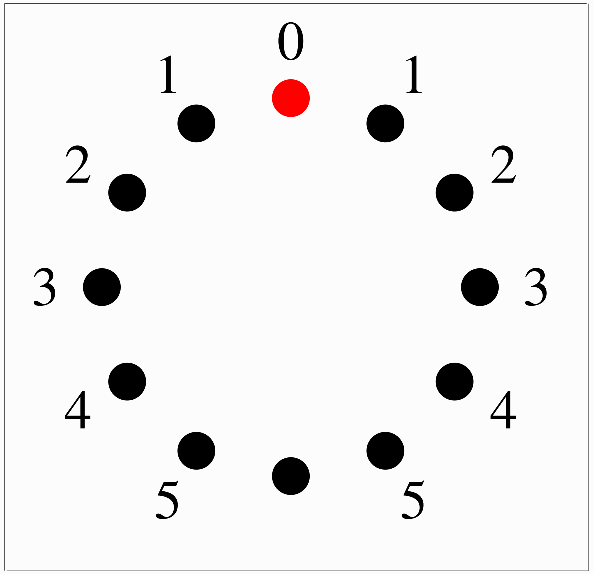

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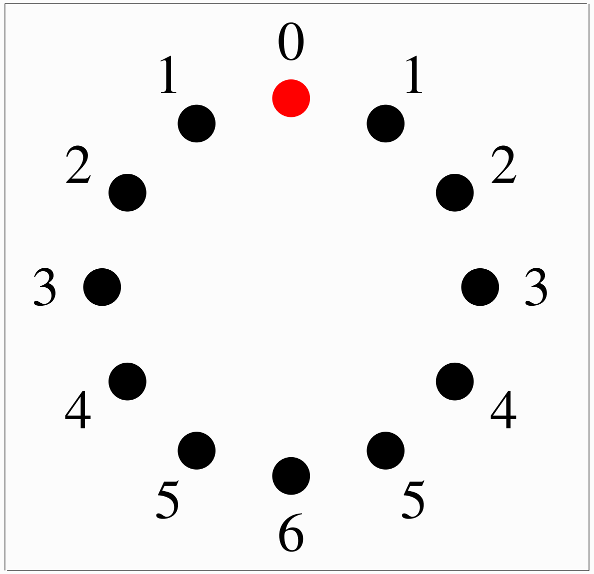

Distance vector and distance type

Replacing intervals by distances we obtain the ***distance vector*** of a tone row.

O. Messiaen's row had interval vector $(5,6,7,7,8,10,8,1,10,6,2,2)$ which yields the distance vector

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A ***distance type*** $(b_1,\ldots,b_6)$ indicates that the distance i occurs exactly b_i times in the distance vector of f.

Consequently the distance type of the last tone row is $(1,4,0,2,3,2)$.

The distance vector is also known as the ***interval class INT-C*** and the distance type as ***Allen Forte's Basic Interval Pattern BIP***.

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Distance types of equivalent tone rows coincide.

All-interval rows have distance type $(2,2,2,2,2,2)$, thus we call them ***all-distances-twice rows***.

All-distances-twice rows

All-interval rows are ***all-distances-twice rows***.

There exist all-distance-twice rows different from all-interval rows.

They have interval types of the form

(1,0,0,1,2,2,0,1,2,2,1), (1,0,0,2,0,2,2,0,2,2,1), (1,0,1,0,1,2,1,2,1,2,1),
(1,0,2,1,2,2,0,1,0,2,1), (1,0,2,2,0,2,2,0,0,2,1), (1,1,0,0,0,2,2,2,2,1,1),
(1,1,0,2,2,2,0,0,2,1,1), (1,1,1,1,1,2,1,1,1,1,1), (1,1,2,0,0,2,2,2,0,1,1),
(1,1,2,2,2,2,0,0,0,1,1), (1,2,0,0,2,2,0,2,2,0,1), (1,2,0,1,0,2,2,1,2,0,1),
(1,2,1,2,1,2,1,0,1,0,1), (1,2,2,0,2,2,0,2,0,0,1), (1,2,2,1,0,2,2,1,0,0,1),
(2,0,0,2,1,2,1,0,2,2,0), (2,0,1,0,2,2,0,2,1,2,0), (2,0,1,1,0,2,2,1,1,2,0),
(2,0,2,2,1,2,1,0,0,2,0), (2,1,0,0,1,2,1,2,2,1,0), (2,1,1,1,2,2,0,1,1,1,0),
(2,1,1,2,0,2,2,0,1,1,0), (2,1,2,0,1,2,1,2,0,1,0), (2,2,0,1,1,2,1,1,2,0,0),
(2,2,1,0,0,2,2,2,1,0,0), (2,2,1,2,2,2,0,0,1,0,0), (2,2,2,1,1,2,1,1,0,0,0)

There exist 4162 non-equivalent all-distances-twice rows.

There exist 519 non-equivalent all-interval rows.

Stabilizer types

Stabilizers of all-interval rows:

- trivial

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- (TI, S^6): the inversion $T \circ I \circ f$ coincides with the shift $f \circ S^6$.
  E.g. $f = (6,3,11,9,8,1,7,10,2,4,5,0)$,
  since $TI = (0,1)(2,11)(3,10)(4,9)(5,8)(6,7)$.
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- (T^6, R): transposing f by 6 pitch classes $T^6 \circ f$ is the same as the retrograde $f \circ R$.
  E.g. $f = (0,3,10,8,7,11,5,1,2,4,9,6)$.

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All-distances-twice rows can also have stabilizers of the form

- (TI, R): the inversion $T \circ I \circ f$ coincides with the retrograde $f \circ R$.
  E.g. $f = (2,7,1,3,4,8,5,9,10,0,6,11)$.

Intervals in acoustic

An interval is a ratio of frequencies. In equal temperament an octave (the frequency ratio $2:1$) is divided into 12 equal parts. Therefore an interval of i semitones corresponds to the ratio $1:2^{i/12}$.

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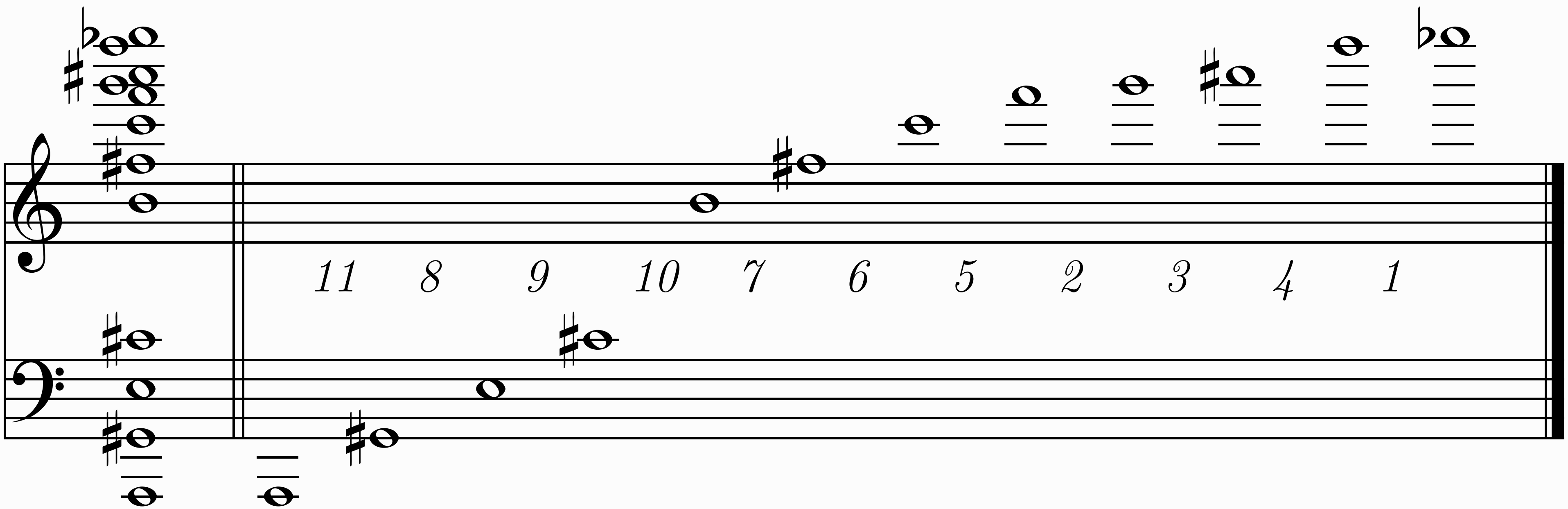
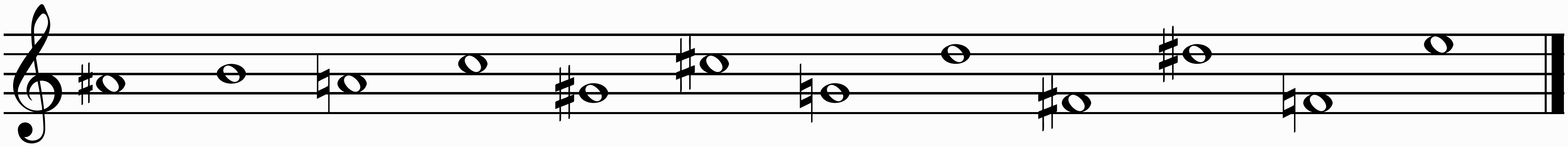
Normally a tone row is not used in a vertical setting.

Fritz Heinrich Klein: Mutter-Akkord (mother-chord), 1919.
It is a chord containing each pitch class and each interval exactly once.



An all-interval row can have a representation, so that all frequency ratios $1 : 2^{i/12}$, $i \in \{1, \ldots, 11\}$ occur



Here we have $+1, -2, +3, -4, +5, -6, +7, -8, +9, -10, +11$.

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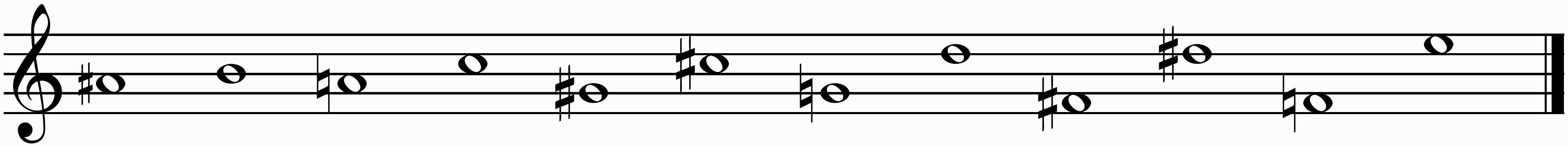
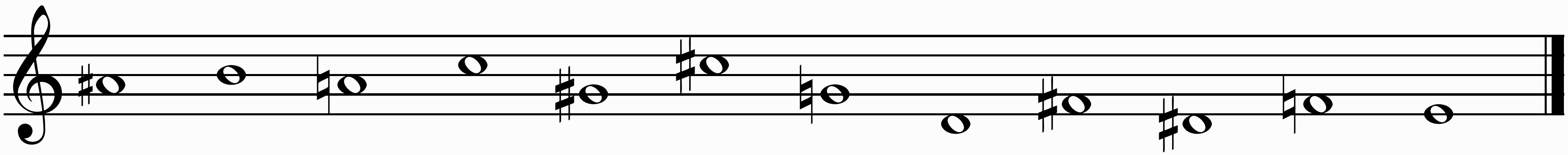


Here we have $+1, -2, +3, -4, +5, -6, +7, -8, +9, -10, +11$.

Now we keep the sequence of pitch classes but change to other representatives with respect to octave equivalence.



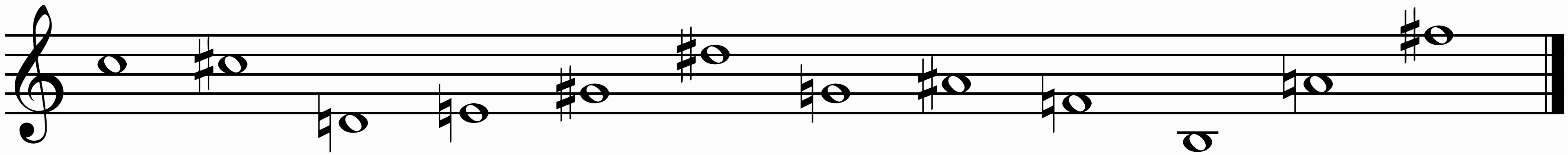
Here we have $+1, -2, +3, -4, +5, -6, -5, +4, -3, +2, -1$.

Now we consider the all-distances-twice row

$f = (0, 1, 2, 4, 8, 3, 7, 10, 5, 11, 9, 6)$

with interval vector $(1, 1, 2, 4, 7, 4, 3, 7, 6, 10, 9, 6)$

and interval type $(2, 1, 1, 2, 0, 2, 2, 8, 1, 1, 0)$.



We hear intervals $+1, -11, +2, +4, +7 - 8, +3, -5, -6, +10, +9$. Thus we hear all possible types of intervals within one octave.

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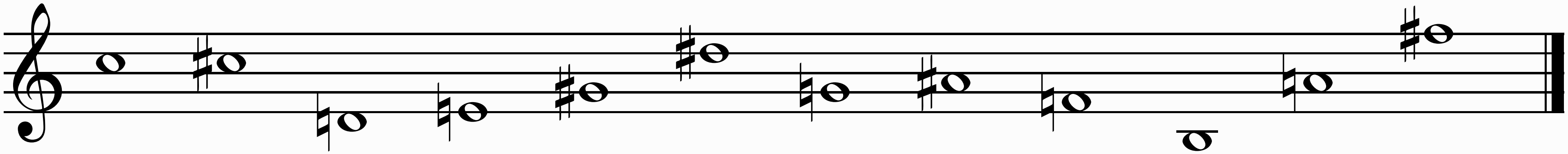
Now we consider the all-distances-twice row
$f = (0,1,2,4,8,3,7,10,5,11,9,6)$
with interval vector $(1,1,2,4,7,4,3,7,6,10,9,6)$
and interval type $(2,1,1,2,0,2,2,8,1,1,0)$.



We hear intervals $+1,-11,+2,+4,+7-8,+3,-5,-6,+10,+9$. Thus we hear all possible types of intervals within one octave.

Possibly Alban Berg's row from the "Lyrische Suite" (1926) was influenced by Klein's mother chord. Maybe that was a reason why all-interval rows were always studied rather as all-interval chords.

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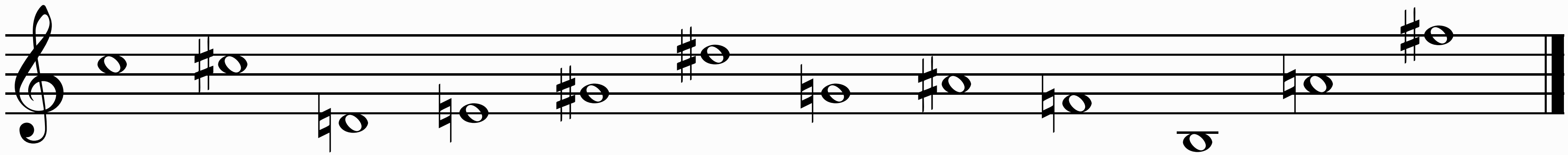

We hear intervals $+1, -11, +2, +4, +7 - 8, +3, -5, -6, +10, +9$. Thus we hear all possible types of intervals within one octave.

Possibly Alban Berg's row from the "Lyrische Suite" (1926) was influenced by Klein's mother chord. Maybe that was a reason why all-interval rows were always studied rather as all-interval chords.

Considering normal usage of a tone row, ***there are no differences between all-interval rows and all-distances-twice rows***.

Remarks on distance types

In total there are 2785 distance types.

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There are two non equivalent tone rows having distance types where only one distance occurs: (12,0,0,0,0,0) and (0,0,0,0,12,0).

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There exist distance types where there is only one equivalence class of tone rows representing this type. E.g. the two examples above or (0,1,0,3,2,6) is represented only by the class of $f = (0,2,8,1,7,3,9,5,11,4,10,6)$.

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$f = (0, 2, 8, 1, 7, 3, 9, 5, 11, 4, 10, 6)$.

The distance type (2,2,2,3,2,1) has 5464 non-equivalent representations as a tone row.

A generalization

Distances correspond to the D_{12}-orbits of 2-sets of pitch-classes: $\{0, 1\}$, $\{0, 2\}$, $\{0, 3\}$, $\{0, 4\}$, $\{0, 5\}$, and $\{0, 6\}$. Thus they correspond to set classes of cardinality 2 in an extension of Allen Fortes "Prime forms and Vectors of Pitch-Class Sets".

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Instead of 2-sets we could consider 3-sets or more general k-sets, $3 \leq k \leq 12$, of consecutive pitch classes in a tone row. Each tone row f determines 12 such sets. We determine the prime form of each set, and obtain a vector of prime forms of k-sets corresponding to f or we could determine the type of prime forms of k-sets of f.

Actually, it is enough to consider $k \leq 6$.

Let us consider the tone row $f = (8, 11, 3, 7, 10, 1, 5, 9, 12, 2, 4, 6)$ of Alban Berg's Violin concerto.

It has the interval vector $(2, 2, 2, 2, 3, 4, 4, 3, 3, 4, 4, 3)$, both interval type and distance type are $(0, 4, 4, 4, 0, 0)$.

There occur 5 different prime forms of pitch classes of cardinality 3.

The vector of prime forms of f is (3,3,3,5,10,11,10,8,10,11,10,5), where 3, 5, 8, 10, and 11 corresponds to

The type of prime forms of 3-sets is therefore $(0, 0, 3, 0, 2, 0, 0, 1, 0, 4, 2, 0)$.

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