E.1.1.2 **Exercise** The ISSN-code ("International Standard Serial Number") is a sequence of eight elements $x_8, \dots, x_1$ taken from the set $\{0, 1, \dots, 9, X\}$. This sequence is divided into two parts, each consisting of four digits, which must be separated by a hyphen. Similar to the ISBN-code, the entries $x_8, \dots, x_2$ are taken from the set $\{0, 1, \dots, 9\}$, and the final entry x_1 is determined such that $\sum_{i=1}^{8} x_i \cdot i \equiv 0 \bmod 11$ is satisfied. If $x_1 = 10$, then x_1 is represented as X. This code has exactly the same properties as the ISBN-code.

1. Determine the ISSN-number of the Bayreuther Mathematische Schriften from the sequence `ISSN 0172-?062` where the digit x_4 is not readable.
2. The number `ISSN 0174-1062` is not a valid ISSN-number. Assuming that exactly one error occurred, determine all valid ISSN-numbers which could be represented by the given one.

E.1.1.3 **Exercise** The EAN-code ("European Article Number") has two basic formats, the 8 and 13 digit variants. The 13 digit code is more common, so we discuss it here. The 8 digit code is generally used where space is restricted. The EAN code is intended as a world wide standard (although some countries use other systems), therefore, no two products may have the same EAN number. To ease the administration of number allocation, each country using EAN has a country identifier at the start of the code. For instance the digits 00 to 13 identify the USA and Canada, 40 to 44 Germany, and 90 to 91 Austria. Other countries have 2 or 3 digit prefixes. The rest of the EAN code is divided into the manufacturer number which can be of variable length, the item reference number, assigned by the manufacturer, and the check digit. In general, both the manufacturer number and the item reference number consist of 5 digits. This means that in this case a manufacturer can have up to 10^5 products. For that reason, those manufacturers which produce a smaller number of products get longer manufacturer codes. The check digit is the last number. All 13 digits $x_{13}, \dots, x_1$ are taken from the set $\{0, 1, \dots, 9\}$. The check digit x_1 is determined by the other digits such that

$$\sum_{i \equiv 1 \bmod 2} x_i + 3 \cdot \sum_{i \equiv 0 \bmod 2} x_i \equiv 0 \bmod 10$$

is satisfied.

1. Show that the EAN-code recognizes a single error and allows the reconstruction of an unreadable entry, but in general it does not detect a swap of two neighboring entries.
2. The EAN of books can easily be obtained from their ISBN-number. As prefix, the three digits 978 are used, regardless of the country in which the

book was published. Then the ISBN-number with the check digit stripped is appended. Finally, the EAN check digit is computed from these 12 digits as described above. Compute the EAN-code of the present book!

The EAN is also coded in a machine-readable version as a barcode. For this purpose, the EAN is encoded as a binary sequence of bars and spaces. A 1 in the code is represented by a *bar section* and a 0 by a *space section.* Consecutive 1's or 0's are combined to form wider bars or spaces. The EAN barcode consists of the following parts:

- The left-hand starting section of the form 101,
- binary encodings of the digits $x_{12}, \ldots, x_7$,
- the center pattern of the form 01010,
- binary encodings of the digits $x_6, \ldots, x_1$,
- the right-hand closing section of the form 101.

For the encoding of $x_{12}, \ldots, x_1$ three different codes are used, codes A, B, and C. (See also [104, 1.2.5 Beispiel].) The codes A and B are applied for encoding $x_{12}, \ldots, x_7$, and code C is used for encoding $x_6, \ldots, x_1$. So far the digit x_{13} has not been encoded. Depending on the value of x_{13}, different sequences of the codes A and B are applied for the encoding of $x_{12}, \ldots, x_7$. They are given in the table below:

x_{13}	x_{12}	x_{11}	x_{10}	x_9	x_8	x_7
0	A	A	A	A	A	A
1	A	A	B	A	B	B
2	A	A	B	B	A	B
3	A	A	B	B	B	A
4	A	B	A	A	B	B
5	A	B	B	A	A	B
6	A	B	B	B	A	A
7	A	B	A	B	A	B
8	A	B	A	B	B	A
9	A	B	B	A	B	A

digit	code A	code B	code C
0	0001101	0100111	1110010
1	0011001	0110011	1100110
2	0010011	0011011	1101100
3	0111101	0100001	1000010
4	0100011	0011101	1011100
5	0110001	0111001	1001110
6	0101111	0000101	1010000
7	0111011	0010001	1000100
8	0110111	0001001	1001000
9	0001011	0010111	1110100

We realize that for encoding x_{12}, code A is always used. If $x_{13} = 0$ then all digits $x_{12}, \ldots, x_7$ are encoded with code A. In all other cases, codes A and B are each used to encode three digits.

The three codes A, B, and C encode each digit as a binary word of length 7. Each codeword consists of two bar and two space sections. No bar or space is longer than four elements. All codewords of codes A and B start with 0 and end with 1. All codewords of code C start with 1 and end with 0. Actually, it would be enough to describe the codewords of code A. In order to obtain the codewords of code C from code A, exchange the 0's and 1's in the codewords of A. In order to obtain the codewords of code B from code C, reverse the order of each codeword of C.

Moreover, we realize that no codeword occurs in two different codes, and no codeword of A is the reverse of a codeword in C. This fact, together with the rule that x_{12} is always encoded with code A allows the determination of the direction (from left to right or from right to left) in which a barcode is read. When reading from left to right, after the left-hand starting section 101, the reader comes across an element of code A. When reading from right to left, after the reverse of the right-hand closing section, which is again 101, the reader comes across the reverse of an element of code C. Consequently, after reading the first digit it is possible to determine the direction of reading.

Finally, let us consider the following example. The book *Codierungstheorie, Springer, Berlin, 1998,* by some of the present authors and K.-H. Zimmermann, has the `ISBN 3-540-64502-0`. First, this number is encoded as an EAN of the form 9783540645023 where the last 3 is the EAN check digit. Since $x_{13} = 9$, we have to apply the codes $ABBABA$ for the encoding of $x_{12}, \ldots, x_7$. This way we obtain the following binary representation of the bar code of 9783540645023.

101	left-hand starting
0111011 0001001 0100001 0110001 0011101 0001101	$x_{12} \ldots x_7$
01010	center pattern
1010000 1011100 1001110 1110010 1101100 1000010	$x_6 \ldots x_1$
101	right-hand closing

This gives a barcode of the form:

9 783540 645023