Cellular basis for generalized blob algebras

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Abstract

Cellular algebras were introduced by Graham-Lehrer [2] as a general framework for studying modular representation theory. They are finite dimensional algebras endowed with a 'cellular basis' such that the structure constants with respect to this basis satisfy certain natural conditions. A cellular algebra $\mathcal{A}$ is always equipped with a family $\{\Delta(\lambda)\}$ of 'cell modules' for λ running over a poset Λ which is part of the cellular basis data. Each cell module $\Delta(\lambda)$ is endowed with a bilinear form $\langle \cdot, \cdot \rangle$ and all the irreducible $\mathcal{A}$-modules arise as quotients by the radical of the form $L(\lambda) = \Delta(\lambda)/\text{rad}\langle \cdot, \cdot \rangle$. Using this, there is for a cellular algebra $\mathcal{A}$, a concrete way to find the irreducible $\mathcal{A}$-modules.

The original blob algebra $b_n = b_n(q, m)$, also known as the Temperley-Lieb algebra of type B, was introduced by Martin and Saleur in [6] via considerations in statistical mechanics. The algebra b_n is certain quotient of a cyclotomic Hecke algebra $\mathcal{H}_n(q_1, q_2)$ of level $l = 2$.

Martin and Woodcock introduced in [7] the natural generalization $\mathcal{B}_n$ of b_n considering the corresponding quotient of a cyclotomic Hecke algebra $\mathcal{H}_n(q_1, \dots, q_l)$ in any level l. Its representation theory has received a considerable amount of interest in recent years.

We give a concrete construction of a cellular basis for the generalized blob algebra $\mathcal{B}_n$ [5]. Our construction uses the isomorphism between KLR-algebras and cyclotomic Hecke algebras, proved by Brundan-Kleshchev [1].

In this talk I will give a short introduction to cellular bases and its applications to representation theory, then I will provide a diagrammatical presentation of generalized blob algebras as a quotient of KLR-algebras and finally I will explain some of the crucial steps in our construction.

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References

[1] J. Brundan, A. Kleshchev, *Blocks of cyclotomic Hecke algebras and Khovanov-Lauda algebras*, Invent. Math. **178** (2009), 451-484.

[2] J. J. Graham, G. I. Lehrer, *Cellular algebras*, Inventiones Mathematicae **123** (1996), 1-34.

[3] J. Hu, A. Mathas, *Graded cellular bases for the cyclotomic Khovanov-Lauda-Rouquier algebras of type A*, Adv. Math., **225** (2010), 598-642.

[4] M. Khovanov, A. Lauda, *A diagrammatic approach to categorification of quantum groups I*, Represent. Theory **13** (2009).

[5] D. Lobos, S. Ryom-Hansen, *Graded cellular basis and Jucys-Murphy elements for generalized blob algebras*, arXiv:1812. 11143.

[6] P. P. Martin, H. Saleur, *The blob algebra and the periodic Temperley-Lieb algebra*, Lett. Math. Phys. **30** (1994), 189-206.

[7] P. P. Martin, D. Woodcock, *Generalized blob algebras and alcove geometry*, LMS Journal of Computation and Mathematics **6**, (2003), 249-296.

[8] D. Plaza, S. Ryom-Hansen, *Graded cellular bases for Temperley-Lieb algebras of type A and B*, Journal of Algebraic Combinatorics, **40**(1), 137-177.

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