

References: [Pierce]: R.S. Pierce "Associative Algebras", GTM 88  
 [G-W]: K.R. Goodearl, R.B. Warfield "Introduction to noncommutative Noetherian rings", CUP  
 [McR]: McConnell - Robson "Noncommutative Noetherian rings", AMS

① Quaternions: Hamilton:  $A = K \oplus Ki \oplus Kj \oplus Kk$  with mult  $i^2 = -1$   $j^2 = -1$   $ij = -ji = k$   
 §1.6 in [Pierce]: general quaternions

- Lemma:  $(\begin{smallmatrix} a & b \\ 0 & 0 \end{smallmatrix})$  is simple with center  $F \Leftrightarrow a \neq 0, b \neq 0$
- Prop: div. alg.

§1.7 in [Pierce]: isomorphisms of quaternion algebras

② Group algebras (see lecture) [G-W, xvi]

- Representations first part
- Books [Pierce 1.2]: Def, Prop about ext. of morphisms
- Connections to skew Laurent rings?
- [McR]: p22: Generalizations
- Semi-simplicity
- Decomposition: [Serre p48], Center [Serre]

③ Rings of diff. operators xvii in [G-W]

- Def  $\rightsquigarrow$  p26 more detail [G-W]

- Weyl algebra p.30

in part Cor 2.2: all Weyl algebras are simple rings

④ Triangular matrix rings [G-W p4]  $\rightsquigarrow$  whole section  
 $\rightsquigarrow$  From examples: When noetherian?

⑤ Skew polynomial rings

[G-W] p. 9 : Universal property

• Quantized coordin. ring of  $k^2$

• Simplicity p. 20

⑤' Skew Laurent rings

[G-W] p. 15 + 20 +

⑥ Universal enveloping algebra  
( [G-W] xix , Chn 14 of [McC-R] ; Basics )

⑦ Clifford algebras

→ see [Buchwitz-Faber-Inyelt: Magic Square of reflections and rotations]  
arXiv: 1806.04600

Section 8 + References (need tensor algebra for the definition)

↳ examples of real Clifford algebras

⑧ TL-algebras : Survey by [Doty - Giacinto : Origins of the Temperley-Lieb algebra : Early history] arXiv 2307.11929