

- References: [Pierce]: R.S. Pierce "Associative Algebras", GTM 88  
 [G-W]: K.R. Goodearl, R.B. Warfield "Introduction to noncommutative Noetherian rings", CUP  
 [McR]: McConnell - Robson "Noncommutative Noetherian rings", AMS

① Quaternions: Hamilton:  $A = K \oplus Ki \oplus Kj \oplus Kk$  with mult  $i^2 = -1$   $ij = -ji = k$   
 $j^2 = -1$   
 §1.6 in [Pierce]: general quaternions  
 • Lemma:  $(\mathbb{A} \neq \mathbb{B})$  is simple with center  $F \Leftrightarrow \mathbb{A} \neq 0, \mathbb{B} \neq 0$   
 • Prop: div. alg.  
 §1.7 in [Pierce]: isomorphisms of quaternion algebras

② Group algebras (see lecture) [G-W, xvi]  
 • Representations  $\downarrow$  first part  
 • Books [Pierce 1.2]: Def, Prop about ext. of morphisms  
 • Connections to skew Laurent rings?  
 • [McR]: p22: Generalizations  
 • Semi-simplicity  
 • Decomposition: [Serre p48], Center [Serre]

③ Rings of diff. operators xvii in [G-W]  
 • Def  $\rightsquigarrow$  p26 more detail [G-W]  
 • Weyl algebra p.30  
 in part Cor 2.2: all Weyl algebras are simple rings

④ Triangular matrix rings [G-W p4]  $\rightarrow$  whole section  
 $\rightsquigarrow$  From examples: When noetherian?

⑤ Skew polynomial rings  
[G-W] p. 9 : Universal property  
• Quantized coordin. ring of  $k^2$   
• Simplicity p. 20

⑤' Skew Laurent rings  
[G-W] p. 15 + 20 +

⑥ Universal enveloping algebra  
( [G-W] xix , Chn 14 of [McC-R] ; Basics )

⑦ Clifford algebras

→ see [Buchwitz-Faber-Inyelt: Magic Square of reflections and rotations]  
arXiv: 1806.04600

Section 8 + References (need tensor algebra for the definition)  
↳ examples of real Clifford algebras

⑧ TL-algebras : Survey by [Doty - Giacinto : Origins of the Temperley-Lieb algebra: Early history] arXiv 2307.11929