MATH5253: Commutative Algebra and Algebraic Geometry

Lecture: 13:00-15:00 Thu @ Roger Stevens LT02 (7M.02)
12:00–14:00 Fr @ Roger Stevens LT03 (7.03)
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Office Hours: by appointment

Course Webpage: `http://www1.maths.leeds.ac.uk/~pmtemf/teaching.html`
This site has important information regarding homework assignments.

Prerequisites: Familiarity with abstract reasoning and proofs, basics of algebra (MATH3193 or equivalent may be useful). It will also be useful to have an inherent interest in geometry and a desire to challenge yourself with some difficult but very interesting mathematics.

Course Description: Commutative algebra is the study of commutative rings, their ideals, and modules. Its origins lie in the work of eminent mathematicians such as Kronecker, Dedekind, Hilbert and Emmy Noether who sought to develop a solid foundation for Number Theory. Later, the field was enriched by its relation to modern Algebraic Geometry, Topology, Homological Algebra, and Combinatorics.
This course will provide an introduction to commutative algebra and then focus on the beautiful connections to and interplay with algebraic geometry, the study of solution spaces for systems of polynomial equations.
Specific topics covered in this course will include ideals and modules, noetherian rings, localisation, primary decomposition, Noether normalisation and Hilbert's Nullstellensatz, algebraic varieties, algebra-geometry dictionary, projective varieties.

References: We will not follow any particular textbook, but recommended sources include:

- Miles Reid's books *Undergraduate Commutative Algebra* and *Undergraduate Algebraic Geometry*.
- The book *Introduction to commutative algebra* by Atiyah–Macdonald provides an excellent introduction (as well as many exercises!).
- Cox Little O'Shea's Ideals, Varieties, and Algorithms gives a very readable introduction to Gröbner bases as well as the algebra geometry dictionary.
- The book *Algebraic curves* by W. Fulton is an introduction into algebraic geometry by treating the case of algebraic curves.

The following also contain more advanced topics but may also help in understanding the material:

- D. Eisenbud Commutative algebra with a view towards algebraic geometry.
- H. Matsumura Commutative ring theory.
- And not for the faint of heart: R. Hartshorne Algebraic Geometry.

Homework: The best way to learn mathematics is by actually doing it. As such, homework will be assigned regularly in this course (roughly every other week). Problem sets and deadlines will be posted on the webpage and you are expected to check back regularly. I *strongly* encourage that you work on the problem sets with your classmates, but I insist that each student writes up his/her own solutions to the problems. I trust that you will maintain academic integrity in this regard. Remember that anything you hand in with your name at the top needs to be *your own work in your own words*.

The ultimate step in understanding a concept or solution is to be able to clearly explain it. Moreover, clear exposition of difficult concepts is a skill which is invaluable far beyond mathematics.
As such, your homework sets will be marked both upon the correctness of your solution and upon the quality of your exposition. Handing homework in, however, is entirely voluntary, but also strongly encouraged.

Assessment: The final mark of this course depends 100% on a 3 hour final exam.

Disclaimer: I reserve the right to change anything on this syllabus if I feel it will improve the quality of the course. All changes will be announced in class.

Have a great semester!