MAT 412 HOMEWORK 12 – LAST HOMEWORK!

DUE: APRIL 14, 2017 (BEGINNING OF CLASS)

This homework set covers sections 8.1, 8.2, 8.3. References are to Hungerford, 3rd. edition.

Problem 1. This is a continuation of problem 5 and 6 from last homework. Let $G \subseteq \mathrm{GL}_n(\mathbb{R})$ be a group acting on $\mathbb{R}^n$ via $(A, v) \mapsto Av$.

(a) Show that G also acts on $\mathbb{R}[x_1, \ldots, x_n]$ via $(g, f) \mapsto g \cdot f := f(g^{-1}(x_1, \ldots, x_n))$.
(b) In the case $G = S_3$ acting on $\mathbb{R}^3$, find three polynomials of degree $1, 2, 3$ that stay invariant under the group action, that is $\sigma \cdot f = f$ for any $\sigma \in S_3$.

Problem 2. The orbit-stabilizer theorem: Let G be a group acting on a set X. If $|G|$ is finite, then show that $|G| = |G \cdot x||G_x|$ for any $x \in X$. Here $G \cdot x$ denotes the orbit of x and G_x the stabilizer subgroup of G.

Problem 3. (a) Show that a subgroup $H \subseteq G$ with $[G : H] = 2$ is normal in G. (Note: Do not assume that the groups are finite)
(b) Can you generalize the statement of (a) to subroups H of index p, where p is prime? If yes, provide a proof, if no, give a counterexample.

Problem 4. Let G be a group and let S be the set of all elements $aba^{-1}b^{-1}$ for $a, b \in G$. The subgroup $[G, G]$ generated by all elements in S is called the *commutator subgroup of* G.

(a) Show that $[G, G]$ is normal in G and that $G/[G, G]$ is abelian. (8.3.33)
(b) Show that A_n is the commutator subgroup of S_n for $n \geq 3$. [*Hint:* You can use here that A_n is generated by 3-cycles.]