Basic Principles of (P)MRI

or:

How I learned to stop worrying and love k-space

What is not in this talk:

- Physics of MRI
  (spin/magnetic field interaction: just cartoons ...)
- Mathematics of MRI
  (sampling theory, reconstruction algorithms)
- Pretty pictures
  (stolen from all over the internet)

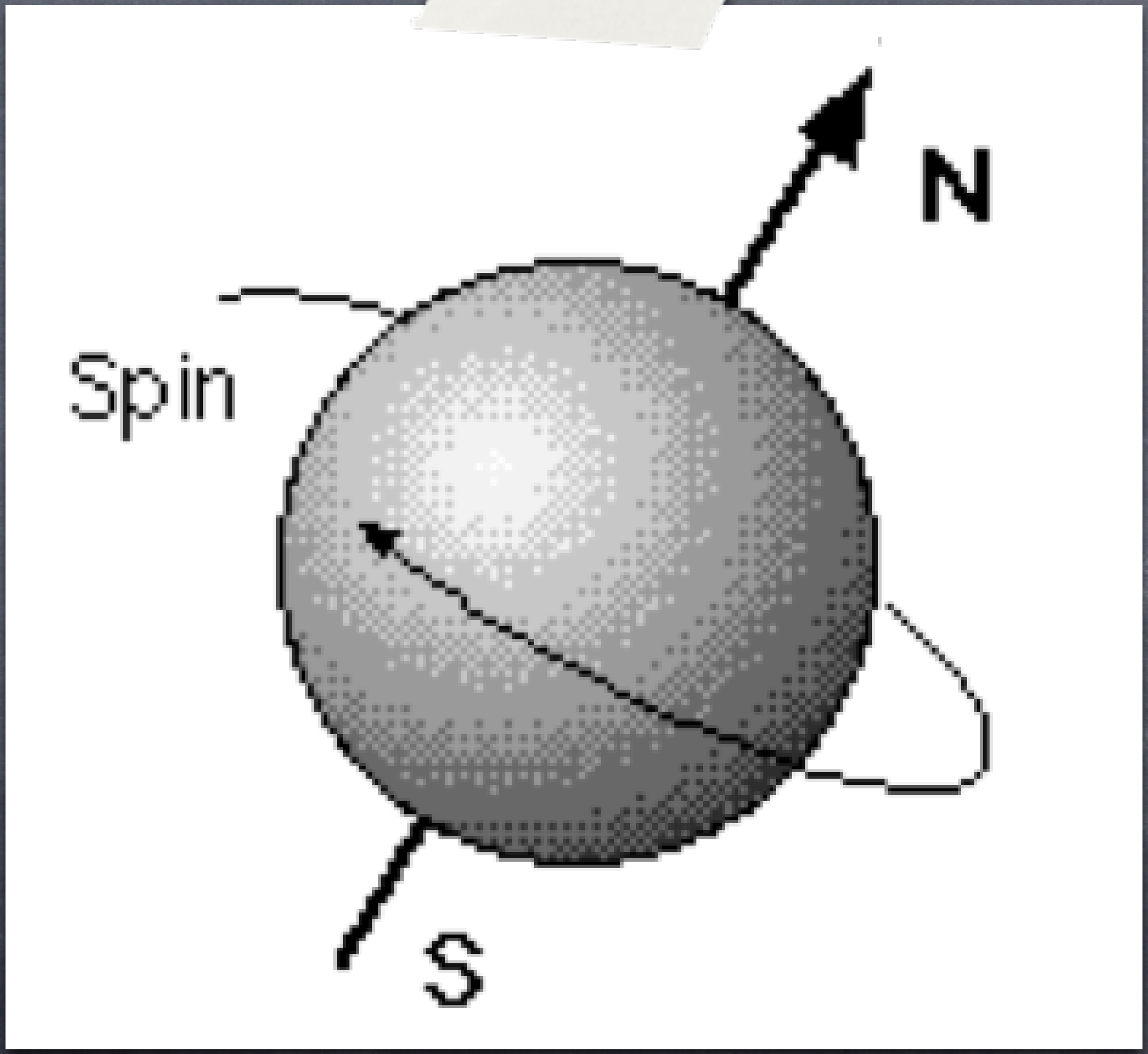
What is in this talk:

- What is really measured in a MRI scan?
- How is the image generated?
- Where does parallel imaging come in?

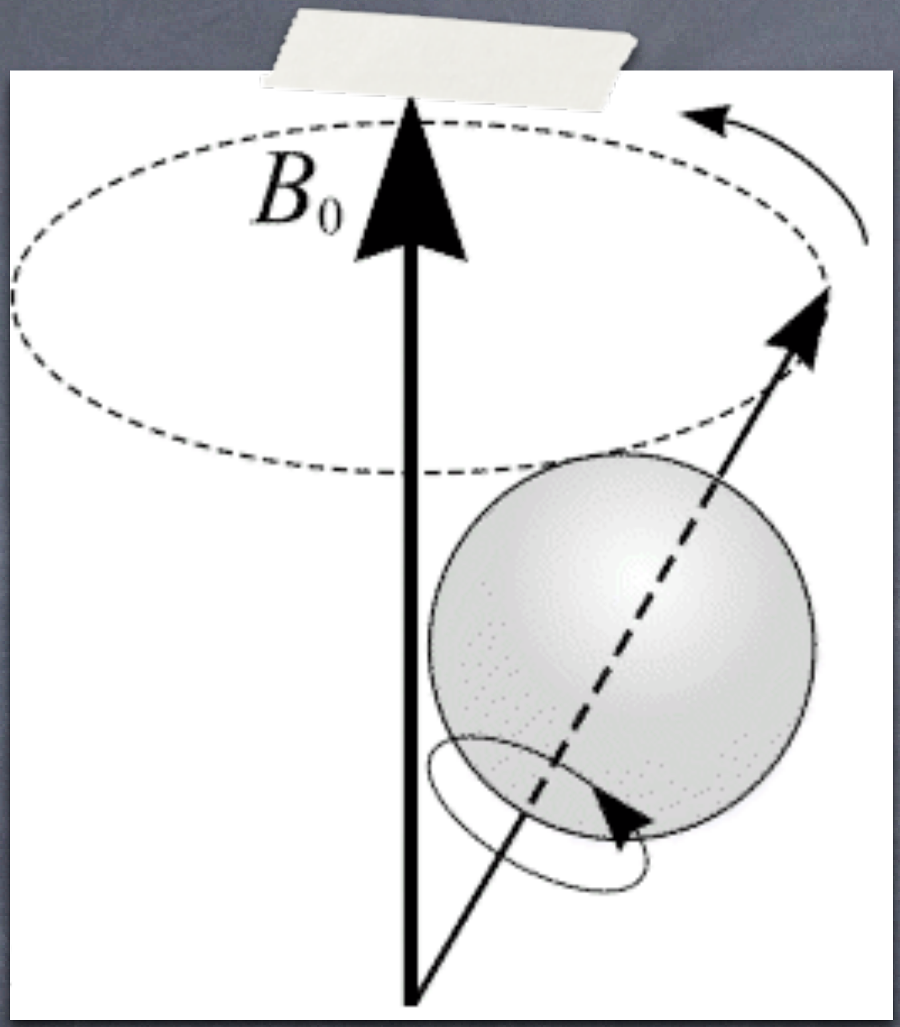
Principles of MRI 1: Signal generation

or: The machine that goes „ping"

MRI is based on nuclear spin of hydrogen



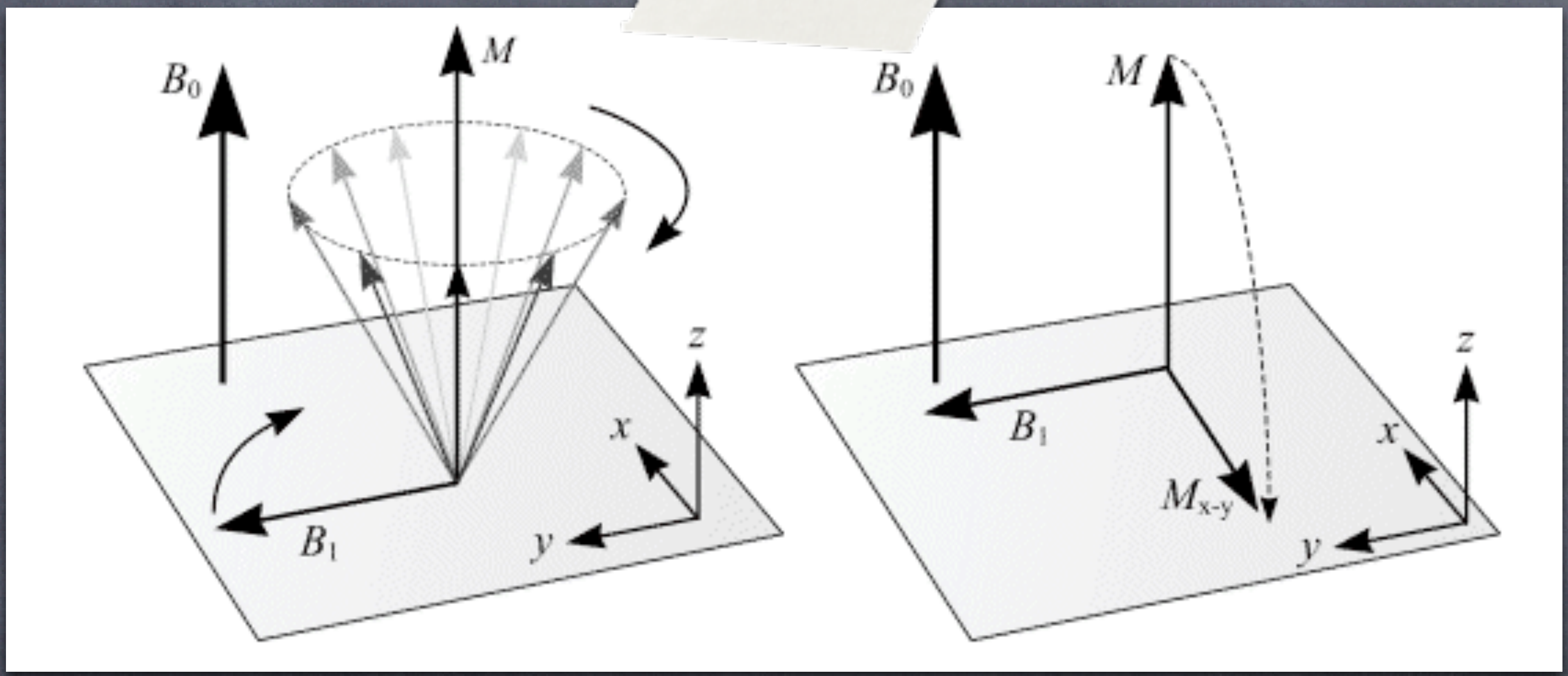
Effect of external magnetic field:



spin axis aligns with field ...

... and precesses

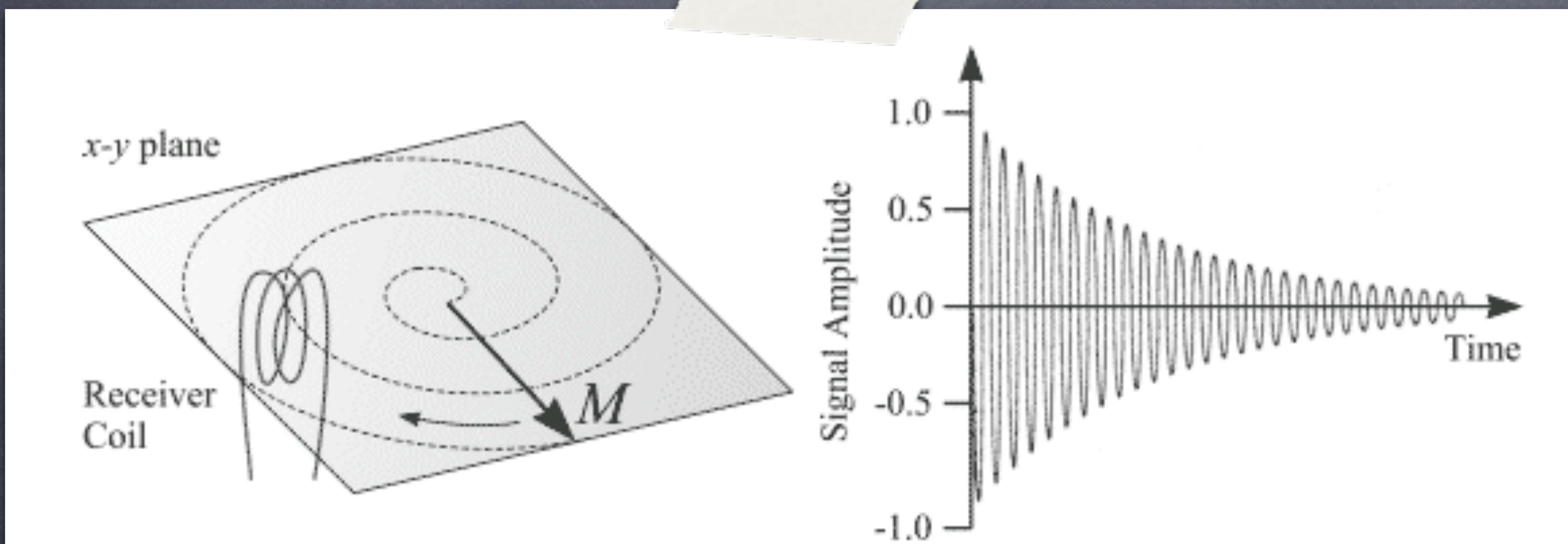
EM pulse at resonance frequency (Larmor frequency)



energy is absorbed,
spin axis …

… flips down

Energy is radiated at Larmor frequency while spin axis realigns with field



rotating magnetic moment induces current …

…this is the measured data

Three fundamental ideas

- Signal strength proportional to hydrogen density
- Larmor frequency proportional to magnetic field strength
- Spin needs time to realign with magnetic field (relaxation time)

Principles of MRI 2: Spatial Encoding

or: „Where did that ping come from?"

Localization of signals

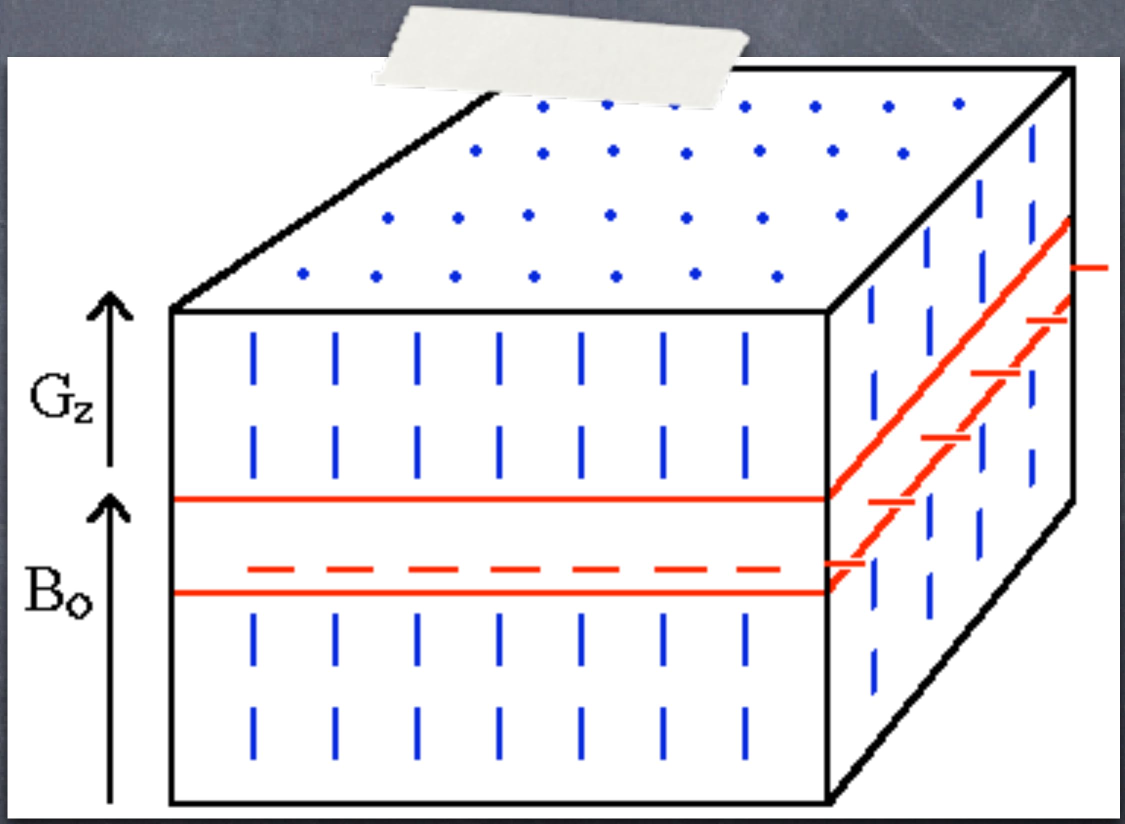
- Goal: spatial map of hydrogen density
- But: Coils measure composite signal from all protons
- Idea: Signal frequency = Larmor frequency depends on magnetic field strength
- <u>Use gradient fields to map signal frequency to spatial location</u>
  (e.g., $G_x = x\ G_0$, $G_y = y\ G_0$, $G_z = z\ G_0$)

Localization of signals

- Problem: Superposition of gradients not unique: $G_x+G_y+G_z=G_0$ at many different points
- One possibility: use many gradient orientations, filtered backprojection (slow)
- In practice: Use gradients sequentially

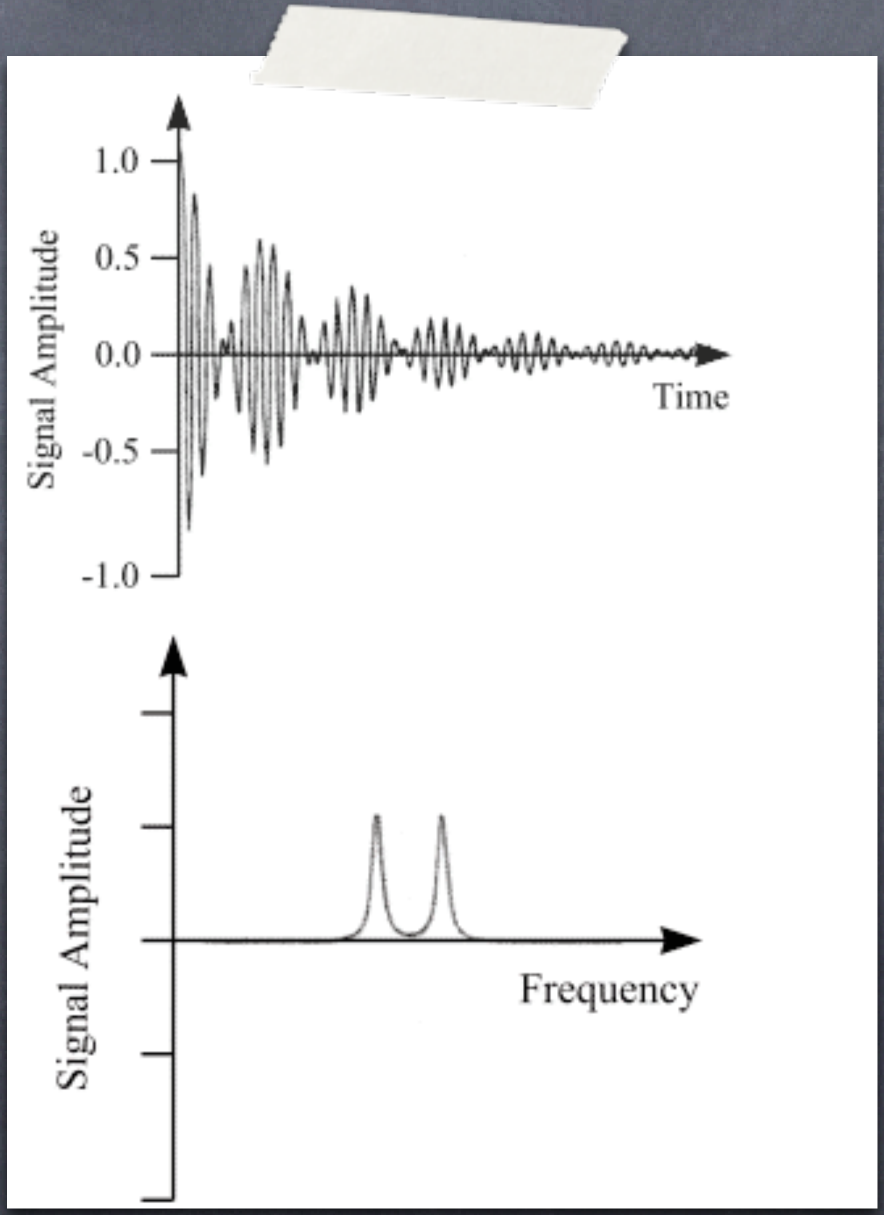
Slice selection (z)

- Use gradient field G_z during excitation
- Only thin slice has resonance at frequency of EM pulse
- Signal comes from single slice only



Frequency encoding (x)

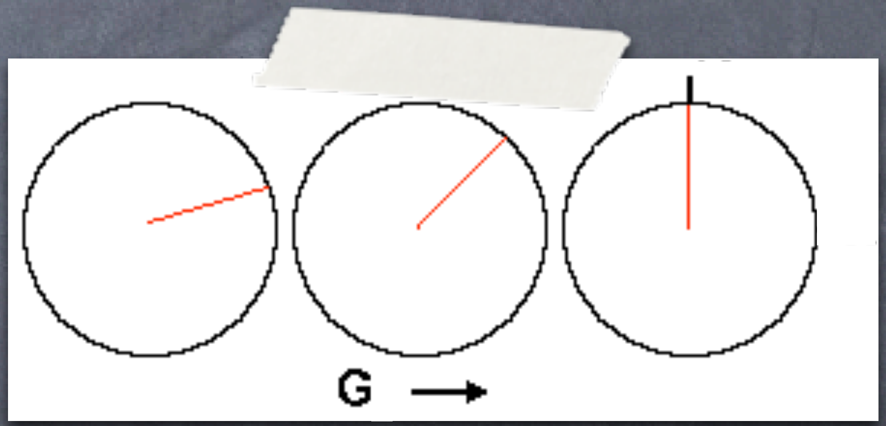
- Use gradient field G_x during measurement
- Resonance frequency differs at each x
- Signal is superposition of frequencies, FT gives contribution of each x position



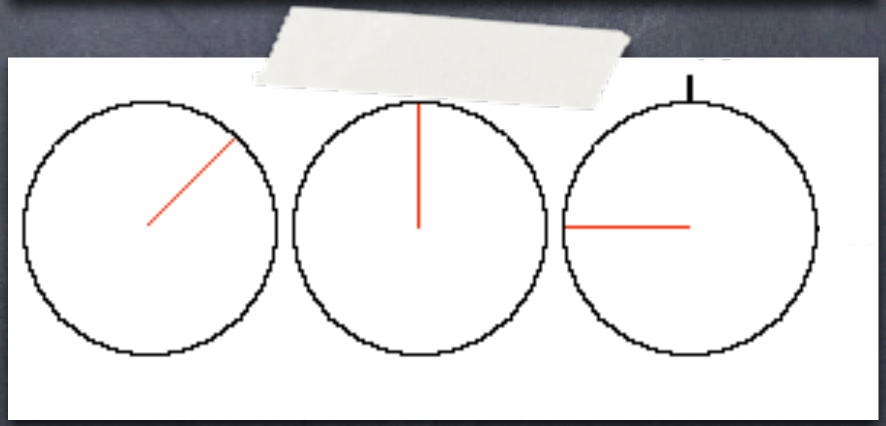
Phase encoding (y)

- Use gradient field G_y to change phase

- Field on: spins precess at different speed



- Field off: same speed, but different phase



Phase encoding (y)

- Problem: phase cannot be measured directly
- But phase difference can!
- Linear gradients: phase difference constant
- ⇒ Repeat measurement with different gradients

  $G_y = n\ y\ G_0,\ n = -N,\ldots,0,\ldots,N$

- Phase difference is y-specific:
  y larger ⇒ gradient change, phase change larger

What is k-space?

- Matrix of measurements
- k_x: time signal
  k_y: diff. phase gradients
- FT in k_x: frequency in time (~ x-position)
  FT in k_y: „frequency" in phase (~ y-position)
- ⇒ like Fourier domain,

  FT gives image

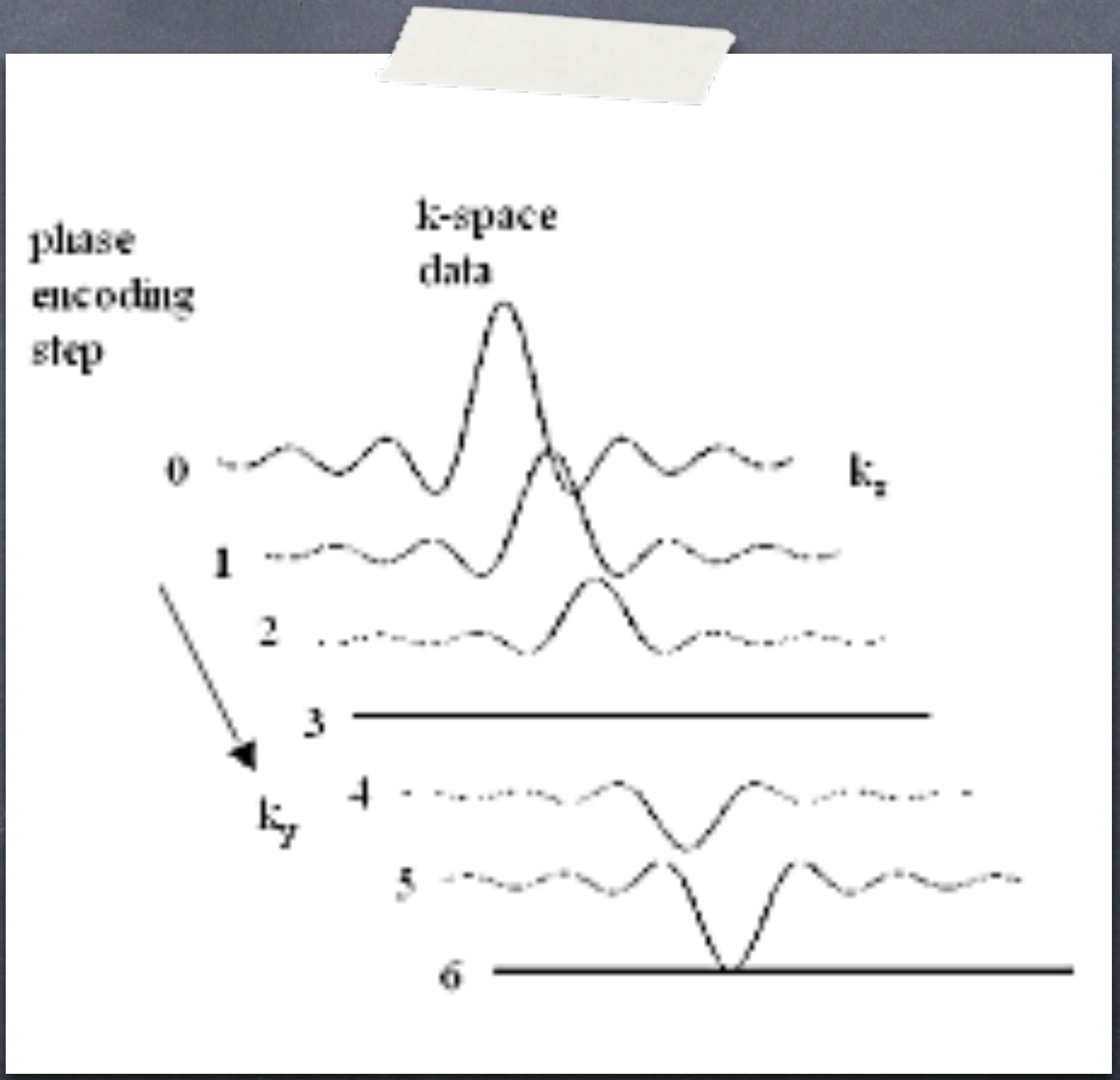


Image reconstruction in MRI

Time Dependent Signals

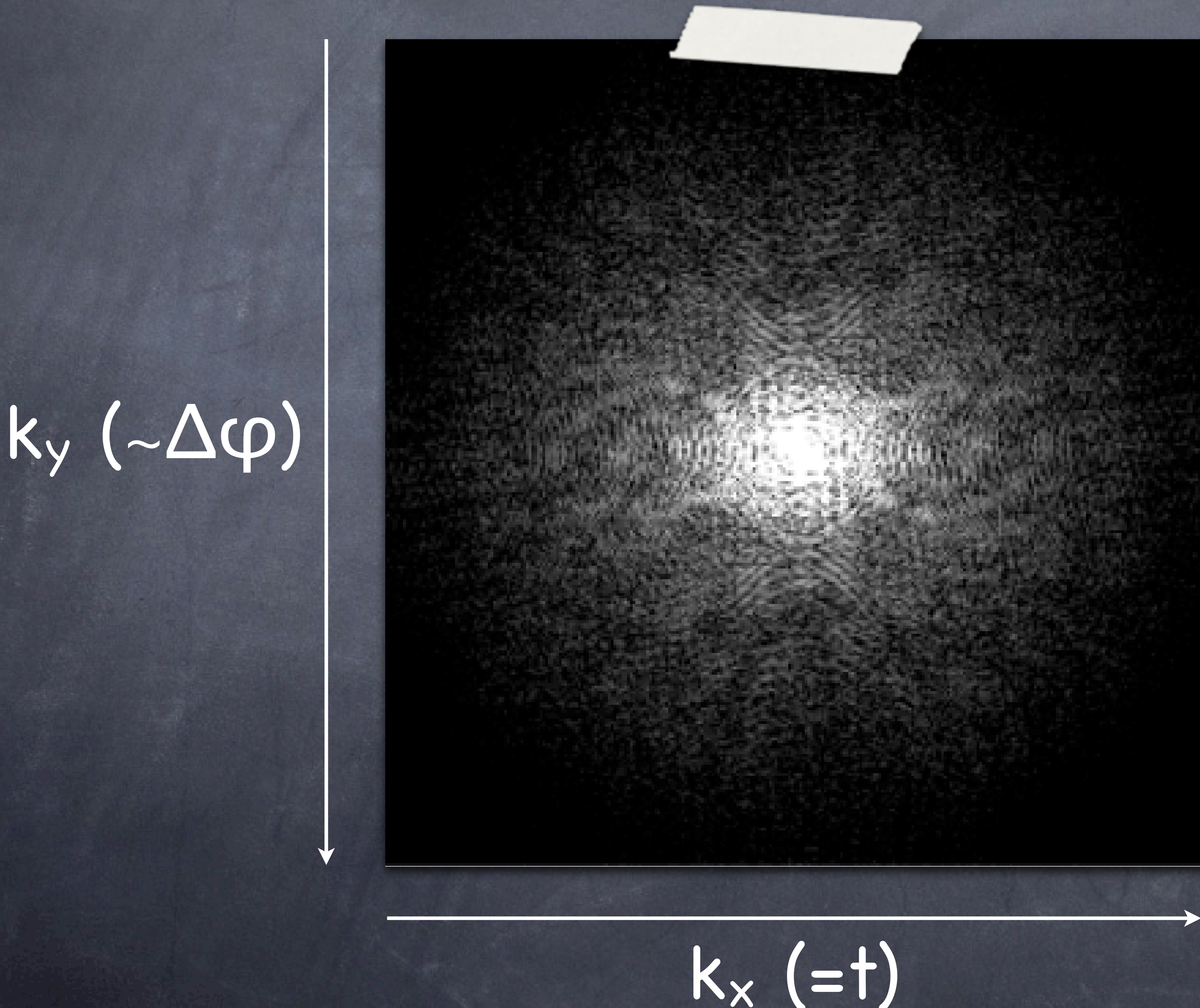
Fourier Transform

Frequency and Phase of Signal

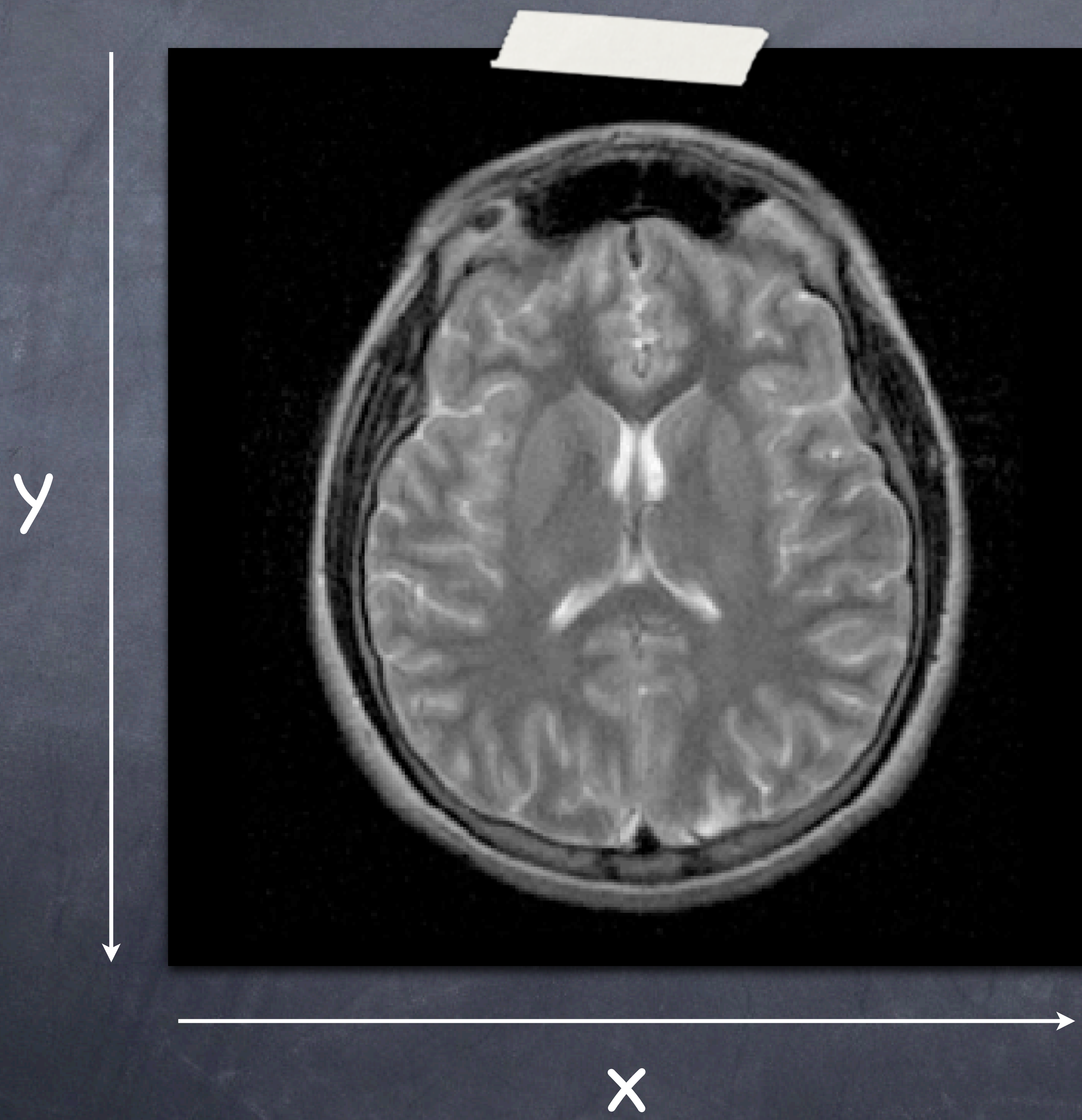
Spatial Encoding

Image

Measured K-Space Data …



… Fourier transformed



Basic steps in MRI scan

1. External field B_0
2. Slice selection: G_z
3. Apply RF pulse
4. turn off G_z
5. Phase encoding: G_y
6. turn off G_y
7. Frequency encoding: G_x
8. Measure signal
9. turn off G_x
10. After relaxation time repeat steps 5-9 with different G_y

Principles of MRI 3: Parallel Imaging

or: „I can`t hold still that long!"

Speed limitations

- Need to turn on and off strong magnetic fields repeatedly
  (fun fact: that's why an MR scan is so loud)
- Need to wait for the spin to realign with mag. field before each phase encoding step
- ⇒ do less phase encoding steps

  (e.g., half the number with double step size)

Aliasing artefacts

- But: equivalent to subsampling in y
- 2π periodic image now 2π/2 periodic
- ⇒ „Wrap around“, size halved in y

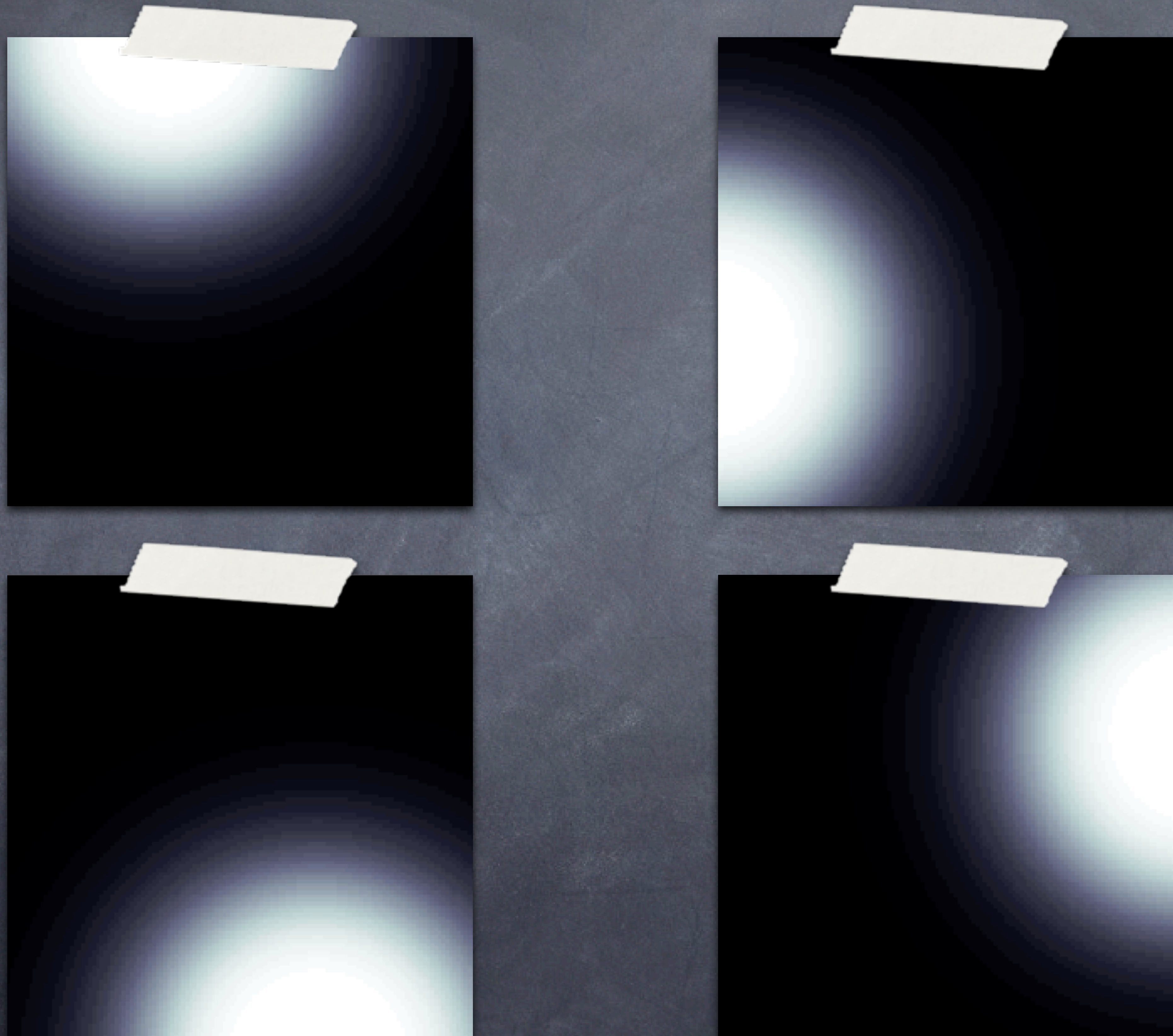
What to do about it

- Information is missing $\Rightarrow$ get more!
- Additional measurements from different coils
- Coils should be independent, e.g., have different spatial sensitivity

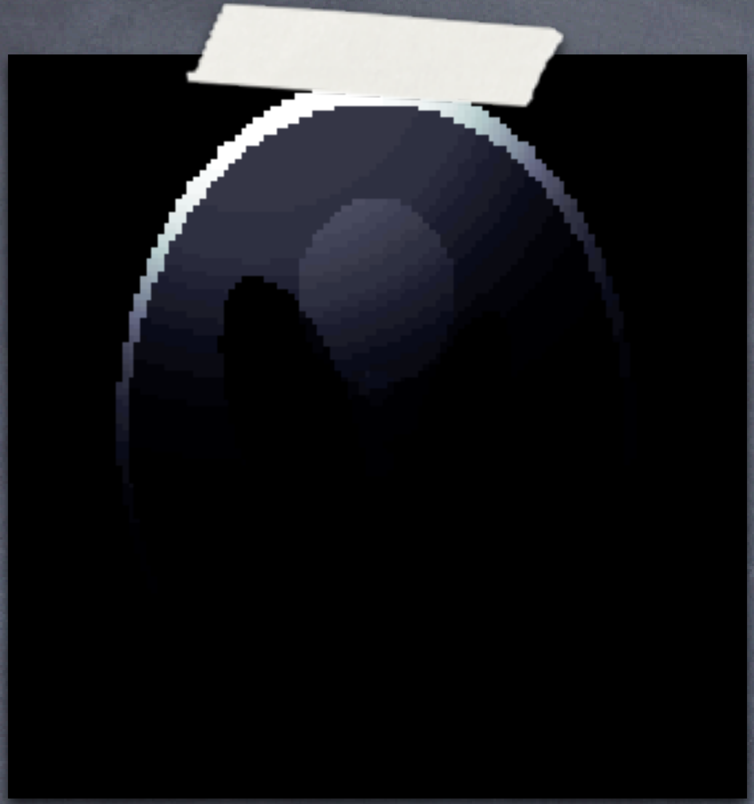
Modeling of Coil Sensitivities

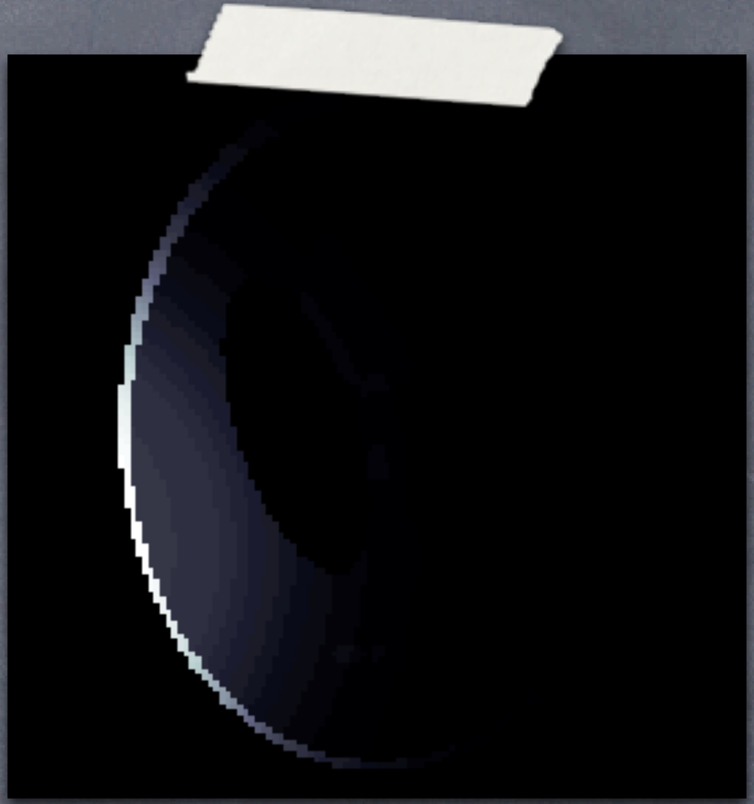
- Sensitivity $S(x,y)$:
  Magnitude of current through coil induced by magnetic moment of unit strength at (x,y)
- Biot-Savart law: Simplification of Maxwell eq's
- Biot-Savart describes magnetic field around current carrying conductor
- By reciprocity: B.-S. describes current induced by magnetic moment (rotating spin axis)

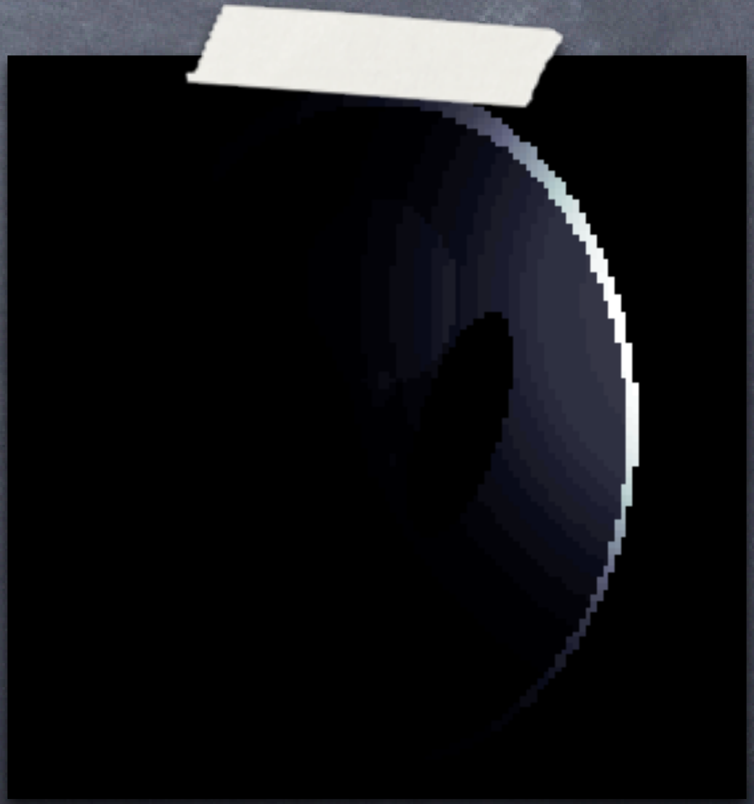
Examples of sensitivities



Examples of sensitivities







Reconstruction: SENSE

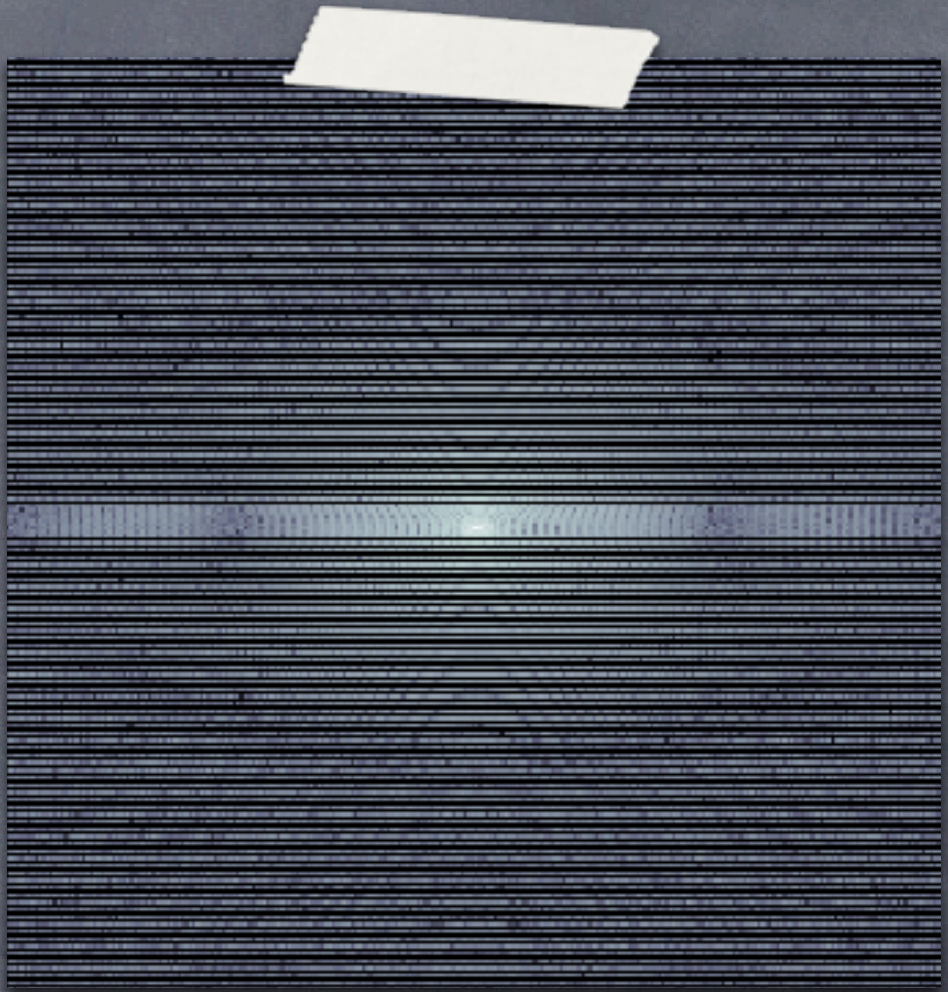
- Measured pixel I(x,y) is superposition of two modulated pixels from true image u
- Sensitivities S_1, S_2 (approximately) known: solve

$$I_1(x,y) = S_1(x,y)u(x,y) + S_1(x,y+\Delta y)u(x,y+\Delta y)$$
$$I_2(x,y) = S_2(x,y)u(x,y) + S_2(x,y+\Delta y)u(x,y+\Delta y)$$

- Overdetermined, model errors: use pseudoinverse

Center Lines

- k-space like Fourier domain: central frequencies most important
- do more phase encoding steps for small n
- ⇒ „weighted wrap around“



Reconstruction: GRAPPA

- Interpolation in k-space using center lines
- Not acquired coefficients are linear combination of measured coefficients
- Compute interpolation weights from center lines
- Enough center lines fully describe sensitivities: GRAPPA equivalent to SENSE

What I didn‘t tell you

- Different imaging types
  (based on other properties such as relaxation time …)
- Pulse sequence design
  (because timing matters …)
- Different sampling patterns
  (radial, spiral, random …)
- Time dependent measurements
  (and other advanced uses such as functional MRI)

If you want to learn more

- Z.-P. Liang, E.M. Haacke,
  Magnetic Resonance Imaging, Encyclopedia of Electrical and Electronics Engineering

- D. Weishaupt, V.D. Koechli, B. Marincek,
  How does MRI work? An Introduction to the Physics and Function of Magnetic Resonance Imaging, 2nd edition, Springer, 2006

Mathematician‘s Summary

- MRI scanner measures Fourier coefficients of image
- Which coeff‘s: determined by magnetic fields, not receiver (same for all coils!)
- PMRI faster by leaving out lines of coeff‘s
- Coil sensitivities are smooth modulations, depend on coil geometry and magnetic properties of sample