

## INTRODUCTION

It has been shown recently that it is possible to achieve superior image quality in Cartesian autocalibrated parallel imaging by reformulating image reconstruction as a nonlinear inversion problem [1,2]. Using this formulation, it is possible to jointly optimize both, the estimated coil sensitivities and the reconstructed image. This leads to a better elimination of undersampling artifacts, due to improved estimations of the coil sensitivity profiles. This approach can also be generalized to arbitrary non-Cartesian sampling patterns. In this work, the performance of nonlinear inversion is investigated for radial sampling, which has the advantage of inherent oversampling of the central part of k-space. This eliminates the need to collect additional reference lines for the estimation of the coil sensitivities, and enables the use of even higher acceleration factors.

## THEORY

We pose the autocalibrated parallel imaging problem as a nonlinear operator equation  $F(x) = g$ , where

$$F : x := (u, c_1, \dots, c_N) \mapsto (K(u \cdot c_1), \dots, K(u \cdot c_N))$$

maps the true image  $u$  and the sensitivities  $c_1, \dots, c_N$  of the  $N$  receiver coils to the set of acquired k-space coefficients  $g = (g_1, \dots, g_N)$  of each coil. The dot denotes pointwise multiplication in image space, and  $K$  is the sampling operator determined by the selected trajectory. This nonlinear equation can be solved by an Iteratively Regularized Gauß-Newton (IRGN) method, i.e., computing in each step  $k$  the minimum  $\delta x$  of

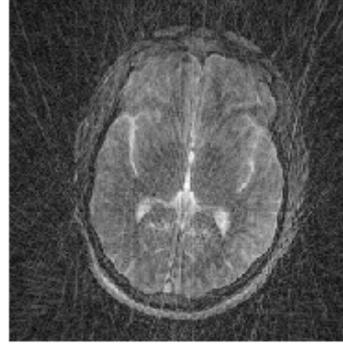
$$\frac{1}{2} \|F'(x_k) \delta x + F(x_k) - g\|^2 + \frac{\alpha_k}{2} \|W(x_k + \delta x - x_0)\|^2$$

for given  $\alpha_k > 0$  and an initial guess  $x_0$ , and then setting  $x_{k+1} := x_k + \delta x$ ,  $\alpha_{k+1} := q \alpha_k$  with  $0 < q < 1$ . Here,  $F'(x_k)$  is the Jacobian of  $F$  evaluated at  $x_k$ , and  $W$  is an operator such that the high Fourier coefficients of the sensitivity components of  $x$  are penalized. Due to the bilinear structure of  $F$ , the action of  $F'(x_k)$  (and its adjoint) can be explicitly calculated using the non-uniform FFT and its inverse [3], so that the corresponding normal equations can be efficiently solved with a conjugate gradient method. It is crucial for numerical stability of the method to replace  $x$  with  $Wx$  and  $F$  by  $FW^{-1}$  instead of applying  $W$  in every iteration, adapting the derivatives accordingly. In this way, instead of applying the weighting operator which is ill-posed, we iteratively invert the inverse operator, which acts as a regularization method. Since any solution of the bilinear equation (1) can only be unique up to reciprocal factors in image and sensitivities, we remove the remaining bias by multiplying the reconstructed image with the sum of squares of the reconstructed sensitivities.

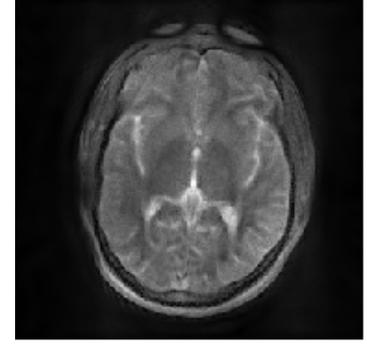
## METHODS AND RESULTS

We acquired a k-space data set of the brain of a healthy volunteer using a radial gradient echo sequence on a clinical 3T scanner with a 4 channel head coil. Sequence parameters were TR=142ms, TE=2.2ms, FA=60°, matrix size (x,y)=256x256, in-plane resolution=0.9mmx0.9mm, slice thickness=5mm. 32 radial projections were acquired for each slice. This corresponds to an undersampling factor of approximately 12 in comparison to a fully sampled radial data set (256· $\pi/2$  projections). Raw data was exported and reconstructed offline using a Matlab (The MathWorks, Natick, MA) implementation of the proposed algorithm.

(a)



(b)



**Fig. 1:** In Vivo brain scan, 256x256 matrix, 32 radial projections, corresponding to an acceleration factor of approximately 12 below the Nyquist limit.

(a) Conventional regridding reconstruction and SOS coil combination.

(b) Proposed non-Cartesian IRGN method.

To make the parameter choice more robust, we set the parameters in relation to the norm of the residual  $n_k$  in step  $k$ . Specifically, we choose  $\alpha_0$  such that  $n_1 \approx 0.75 \cdot n_0$ , and  $q$  such that  $n_2 \approx 1/3 \cdot n_1$ . We stop the iteration in step  $k$  if  $n_k > 1/2 \cdot n_{k-1}$ . Images were also reconstructed with a conventional regridding algorithm, followed by a Sum of Squares (SOS) combination of the individual coil images  $u \cdot c_1, \dots, u \cdot c_N$ . A comparison of both reconstructions for a single slice can be seen in **Fig. 1**.

## DISCUSSION

With the non-Cartesian IRGN method, it is possible to use very high acceleration factors, which allows the acquisition of data sets with high spatial and temporal resolution. Undersampling artifacts, which significantly deteriorated image quality in the conventional regridding reconstruction, were removed efficiently. While no acceleration in z direction was used in this study, the proposed method can be extended to arbitrary 3D sampling patterns by appropriate definition of the sampling operator  $K$ . This has the potential for even higher acceleration factors. It should be noted that our heuristic stopping criterion may not give optimal image quality for different types of data sets. With the current Matlab implementation, the reconstruction time for a single slice of this k-space dataset was approximately 40 seconds on a 2.4 GHz workstation (using only a single core) with 4GB RAM. It is possible to significantly accelerate the reconstruction with a more optimized implementation e.g. by using the high potential of parallel computing on graphics hardware.

## REFERENCES

- [1] Uecker et al., MRM 60:674-682 (2008)
- [2] Ying et al., MRM 57:1196-1202 (2007)
- [3] Fessler et al., IEEE T-SP, 51(2):560-74 (2003)

Acknowledgements: SFB F3209-18