Workshop on
"Geometry and Representation Theory"
January 16 – 27, 2017

organized by

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Week 2
January 23 – 27, 2017

- **Monday, January 23, 2017**

  08:30 – 09:00 **Opening & Registration**

  09:00 – 09:55 **Laurent Manivel**
  Title: New constructions of Calabi-Yau varieties.
  Abstract: I will describe new constructions of Calabi-Yau varieties, as generalized degeneracy loci of morphisms between vector bundles. The universal models of these generalized degeneracy loci will be orbit closures in prehomogeneous vector spaces.

  09:55 – 10:25 *coffee / tea break*

  10:25 – 10:55 **Laura Rider**
  Title: Tilting Modules and the centralizer of a regular nilpotent.
  Abstract: In this talk, Ill discuss the representation theory of a reductive group in positive characteristic with an emphasis on the role played by tilting modules. We will consider their restriction to the (graded) centralizer of a regular nilpotent, and two geometric constructions of this restriction functor: one via the geometric Satake equivalence and another employing exotic coherent sheaves on the Springer resolution.

  11:00 – 11:55 **Wolfgang Sörgel**
  Title: Equivariant motives in representation theory.
  Abstract: I want to explain how recent advances in the theory of motives allow to better understand the phenomenon of gradings on categories of representations. (Joint with M. Wendt and R.Virk.)

  11:55 – 14:00 *lunch break*

  14:00 – 14:55 **Reimundo Heluani**
  Title: Rationality of vertex algebras and higher chiral homology.
  Abstract: I will discuss a question of Beilinson and Drinfeld regarding the vanishing of the higher chiral homology of V (the integrable quotient of the affine Kac-Moody vertex algebra at positive integral level). In particular I will describe an approach (joint with J. Van Ekeren) that relates this question in the particular case of Elliptic curves to classical homological constructions on the Zhu algebra of V. Best.

  15:00 – 15:30 **Ronan Terpereau**
  Title: A symplectic version of the Chevalley restriction theorem.
  Abstract: Let G be a connected reductive linear algebraic group and let V be a visible polar representation of G with Cartan subspace $\mathfrak{c}$ and Weyl group W. It is expected that the symplectic double $(V \oplus V^*)/\!\!/\!\!/ G$ identifies with $(\mathfrak{c} \oplus \mathfrak{c}^*)/W$ as a (symplectic) variety. In this talk I will give some examples and explain briefly what is known about this conjecture.

  15:30 – 15:50 *break*

15:50 – 16:45 Catharina Stroppel

Title: R-matrices and convolution algebras for Grassmannians.

Abstract: In this talk I will describe in detail the combinatorics of equivariant cohomologies of Grassmannians, their Schubert classes (depending on a choice of Borel) and torus fixed point bases and connect them with the representation theory of Lie algebras. In particular we will construct R-matrices as a base change from one choice of Schubert class basis to another. In this way we will construct certain Bethe algebras which are important and interesting from the theory of integrable systems and Bethe bases. Finally we will connect this with the current algebra for $\mathfrak{sl}_2$ and the group algebra of the affine Weyl group and compare it briefly with constructions of Ginzburg, Maulik and Okounkov in a related but different setting.

- **Tuesday, January 24, 2017**

09:00 – 09:55 Drazen Adamović

Title: Conformal embeddings of affine vertex algebras in W-algebras ant their applications.

Abstract: We study conformal embeddings of affine vertex algebras in W-algebras. We classify conformal levels and present results on explicit decompositions of W-algebras as modules for affine vertex algebras. A particular emphasis is given on the application of affine fusion rules to the determination of branching rules. As an application we construct a new family of simple current modules and simple current extensions of affine vertex algebras. We also present explicit realizations of certain W-algebras related to logarithmic conformal field theory. (This talk is based on joint papers with V. G. Kac, P. Moseneder Frajria, P. Papi and O. Perse).

09:55 – 10:25 *coffee / tea break*

10:25 – 10:55 Jethro van Ekeren

Title: Modularity of Relatively Rational Vertex Algebras.

Abstract: I will talk about joint work with T. Arakawa in which we prove a modular invariance theorem on the characters of a vertex algebra equipped with a family of conformal structures. Working in families in this way permits one, roughly speaking, to obtain results under finiteness conditions on the algebra that are weaker than are typically necessary. We apply our theorem to the affine Kac-Moody vertex algebras at admissible level and, via quantised Drinfeld-Sokolov reduction, we deduce the modular transformations of torus one point functions for regular affine W-algebras.

11:00 – 11:55 Alberto de Sole

Title: W-algebras in type A.

Abstract: We describe a method for constructing the generators, and their commutation relations, for the W-algebras of type A. We also see how the analogue result in the classical affine case can be used to construct integrable Hamiltonian hierarchies of Lax type.

11:55 – 14:00 *lunch break*

14:00 – 14:55 Andrew Linshaw

Title: Orbifolds and Cosets of W-algebras.

Abstract: The orbifold and coset constructions are standard ways to create new vertex algebras from old ones. It is believed that orbifolds and cosets will inherit nice properties such as strong finite generation, C_2-cofiniteness, and rationality, but few general results of this kind are known. I will discuss how ideas from classical invariant theory can be used to study orbifolds and cosets of W-algebras. This is based partly on joint work with T. Arakawa, T. Creutzig, and K. Kawasetsu.

15:00 – 15:30 Kazuya Kawasetsu

Title: Quasi-lisse vertex algebras and modular linear differential equations.

Abstract: In this talk, we introduce a notion of quasi-lisse vertex algebras, which generalizes admissible affine vertex algebras. We show that the normalized character of an ordinary module over a quasi-lisse vertex operator algebra has a modular invariance property, in the sense that it satisfies a modular linear differential equation. As an application we obtain the explicit character formulas of simple affine vertex algebras associated with the Deligne exceptional series at level $-h^\vee/6 - 1$, which express the homogeneous Schur indices of 4d SCFTs studied by Beem, Lemos, Liendo, Peelaers, Rastelli and van Rees, as

quasi-modular forms. This is a joint work with Tomoyuki Arakawa.

15:30 – 15:50 *break*

15:50 – 16:45 **Minoru Wakimoto**
Title: Modular invariance in representation theory of superconformal algebras : the case of big $N = 4$ SCA — joint work with Victor Kac.
Abstract: The characters of integrable (or more generally admissible) representations of an affine Lie superalgebra $\mathfrak{g}$ are mock theta functions, and they can be modified to real analytic modular forms by Zwegers' method. And similarly for representations of the W-algebra associated to $\mathfrak{g}$.
In the case rank($\mathfrak{g}$)=2, the modified mock theta functions are uniquely determined and characterized by the elliptic transformation properties and the second order differential equations. In the higher rank cases, however, the uniqueness of modification cannot necessarily be expected.
In this talk, we discuss about the characters of the big $N = 4$ superconformal algebra obtained from integrable $\hat{D}(2, 1; a)$-modules. In this case there exist two kinds of modifications, and each of them is characterized by holomorphic conditions on some variables of the big $N = 4$ SCA. This talk is a joint work with Victor Kac.

- **Wednesday, January 25, 2017**

09:00 – 09:55 **Nicolas Ressayre**
Title: On the tensor semigroup of an affine Kac-Moody Lie algebra.
Abstract: We are interested in the decomposition of the tensor product of two representations of a symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$. Let P_+ be the set of dominant integral weights. For $\lambda \in P_+$, $L(\lambda)$ denotes the irreducible, integrable, highest weight representation of $\mathfrak{g}$ with highest weight λ. Let $P_{+,\mathbb{Q}}$ be the rational convex cone generated by P_+. Consider the *tensor cone*

$$\Gamma(\mathfrak{g}) := \{(\lambda_1, \lambda_2, \mu) \in P_{+,\mathbb{Q}}^3 \,|\, \exists N > 1 \quad L(N\mu) \subset L(N\lambda_1) \otimes L(N\lambda_2)\}.$$

If $\mathfrak{g}$ is finite dimensional, $\Gamma(\mathfrak{g})$ is a polyhedral convex cone described by an explicit finite list of inequalities. In general, $\Gamma(\mathfrak{g})$ is neither polyhedral, nor closed. In this talk, we describe the closure of $\Gamma(\mathfrak{g})$ by an explicit countable family of linear inequalities, when $\mathfrak{g}$ is untwisted affine. This solves a Brown-Kumar's in this case.

10:00 – 10:30 **Oleksandr Tsymbaliuk**
Title: Shifted Quantum Affine Algebras.
Abstract: In this talk, I will speak about the shifted versions of Yangians and quantum affine algebras as well as their incarnations through geometry of parabolic Laumon spaces, additive/multiplicative slices, and Toda lattice.
I will start by reminding the notion of the shifted Yangian (originally introduced by Brundan-Kleshchev in the $\mathfrak{gl}(n)$ case with a dominant shift and later generalized by Kamnitzer-Webster-Weekes-Yacobi to any simple Lie algebra with an arbitrary shift) as well as the recent work relating these algebras to the Coulomb branches.
In the second half, I will discuss the multiplicative analogue of that story. On the algebraic side this leads to the notion of shifted quantum affine algebras, while on the geometric side we replace cohomology by K-theory and additive slices are replaced by multiplicative slices.
This is a joint project with Michael Finkelberg.

10:30 – 11:00 *coffee / tea break*

11:00 – 11:45 **Short talks by students: Raphael Achet** (*Picard group of the forms of the affine line and of the additive group*), **Laura Fedele** (*Quantum finite W-algebras in type A*), **Naoki Genra** (*W-algebras, Screening operators and Wakimoto representations*), **Jens Eberhardt** (*A Formalism of Mixed Sheaves in Positive Characteristic*), **Veronica Vignoli** (*Integrable Hamiltonian Hierarchies Associated to Classical Affine W-Algebras for* $\mathfrak{gl}_n$).

11:45 – 12:00 *break*

12:00 – 12:35 **Short talks by students: Raeez Lorgat** (*Opers and Quantum Curves*), **Matteo Gardini** (*Quantum Vertex Operators Algebra*), **Masahiro Chihara** (*Langlands duality for characters of finite dimensional representation of simple Lie algebra*), **Jacob Matherne** (*A combinatorial Fourier transform for quiver representation varieties in type A*).

- **Thursday, January 26, 2017**

09:00 – 09:55 **Oksana Yakimova**
Title: Index of a seaweed subalgebra and meander graphs in the classical types (joint with D. Panyushev).

09:55 – 10:25 *coffee / tea break*

10:25 – 10:55 **Iva Halacheva**
Title: A cactus group action on crystals and the monodromy of moduli spaces.
Abstract: Given a complex finite-dimensional semisimple Lie algebra $\mathfrak{g}$, the associated cactus group $J(\mathfrak{g})$ is built out of its Dynkin diagram and is a cousin of the braid group. We discuss two instances of an action of $J(\mathfrak{g})$ on any $\mathfrak{g}$-crystal. The first is combinatorial via Schützenberger involutions, while the second comes from a family of maximal commutative subalgebras of $U(\mathfrak{g})$, known as the shift of argument algebras. They act with simple spectrum on a highest weight, irreducible $\mathfrak{g}$-representation V and are indexed by points in a De Concini-Procesi moduli space, so induce a cover on it for a fixed V. We show that the resulting monodromy action coincides with the combinatorial action of the cactus group on the crystal corresponding to V. An analogous construction for the so-called Gaudin algebras can be related in type A via skew Howe duality.

11:00 – 11:55 **Clelia Pech**
Title: Rational curves on horospherical varieties of Picard rank one.
Abstract: In this talk I will report on joint work in progress with R. Gonzales, N. Perrin and A. Samokhin on rational curves on a family of quasi-homogeneous varieties with horospherical group actions. One of the motivations is to study the quantum cohomology of these spaces, an associative and commutative deformation of their usual cohomology ring whose structure constants are given by counts of rational curves.

Using a classification of horospherical varieties of Picard rank one by B. Pasquier we study their moduli spaces of rational curves and deduce a Chevalley-type formula for the quantum cup product by the hyperplane class. Some of the varieties we consider have particularly well-behaved moduli spaces of stable maps, and in these cases we obtain a more precise description of the quantum cohomology.

11:55 – 14:00 *lunch break*

14:00 – 14:55 **Fedor Malikov**
Title: Strongly homotopy chiral algebroids.
Abstract: We shall define the object appearing in the title and obtain a homotopy sensible classification of such objects over a complete intersection affine algebra. The case of ordinary Picard-Lie algebroids will also be reviewed.

15:00 – 15:30 **Vlad Bavula**
Title: The classical left regular left quotient ring of a ring and its semisimplicity criteria.
Abstract: Let R be a ring, $\mathcal{C}_R$ and $'\mathcal{C}_R$ be the set of regular (i.e., non-zero-divisor) and left regular elements of R, respectively ($\mathcal{C}_R \subseteq {}'\mathcal{C}_R$). Goldie's Theorem (1958, 1960) is a semisimplicity criterion for the classical left quotient ring $Q_{l,cl}(R) := \mathcal{C}_R^{-1}R$. Semisimplicity criteria are given for the classical left regular left quotient ring $'Q_{l,cl}(R) := {}'\mathcal{C}_R^{-1}R$. As a corollary, two new semisimplicity criteria for $Q_{l,cl}(R)$ are obtained (in the spirit of Goldie). Applications are given for the algebra of polynomial integro-differential operators.

15:30 – 15:50 *break*

15:50 – 16:45 **Avraham Aizenbud**
Title: Bounds on multiplicities of spherical spaces over finite fields.
Abstract: Let G be a reductive group scheme of type A acting on a spherical scheme X. We prove that there exists a number M such that the multiplicity $\dim \mathrm{Hom}(\rho, \mathbb{C}[X(F)])$ is bounded by M, for any fini-

te field F and any irreducible representation ρ of $G(F)$. We give an explicit bound for M. We conjecture that this result is true for any reductive group scheme and when F ranges (in addition) over all local fields of characteristic 0.

- **Friday, January 27, 2017**

  09:00 – 09:55 **Stephane Gaussent**
  Title: Kac-Moody groups, Masures and Iwahori-Hecke algebras.
  Abstract: Masures are analogues of Bruhat-Tits buildings in the Kac-Moody setting. They are designed to help the study of Kac-Moody groups over local fields. Using the masure, the spherical Hecke algebra associated to any such group can be defined. In this talk, after discussing the definition of a masure, I will report on a joint work with Nicole Bardy-Panse and Guy Rousseau where we define the Iwahori-Hecke algebra in this context. This algebra might be seen as the affine Iwahori-Hecke algebra associated to the Weyl group of the Kac-Moody group.

  09:55 – 10:25 *coffee / tea break*

  10:25 – 10:55 **Tina Kanstrup**
  Title: Categorical braid group actions.
  Abstract: Let G be a reductive algebraic group. Bezrukavnikov and Riche constructed a categorification of the affine braid group inside the derived category of G-equivariant coherent sheaves on the Steinberg variety. Ben-Zvi and Nadler noticed that this category h as a meaning in the realm of Derived Algebraic Geometry. A model for this interpretation not involving any higher category theory can be constructed using matrix factorizations. The braid group action of Bezrukavnikov-Riche can be transferred to this model. This is joint work with Sergey Arkhipov.

  11:00 – 11:55 **Simon Goodwin**
  Title: Modular W-algebras and reduced enveloping algebras.
  Abstract: We give an overview of joint work with Lewis Topley on modular W-algebras. In particular, we outline the classification 1-dimensional modules for modular W-algebras for $\mathfrak{gl}_n$, which in turn this leads to a classification of minimal dimensional modules for reduced enveloping algebras for $\mathfrak{gl}_n$.

  11:55 – 14:00 *lunch break*

  14:00 – 14:30 **Tanmay Deshpande**
  Title: Character sheaves and modular categories.
  Abstract: Character sheaves on an algebraic group are supposed to be geometric analogues of irreducible characters of finite groups. Now let G be an algebraic group over a finite field $\mathbb{F}_q$. It seems that the relationship between character sheaves on G and irreducible characters of the finite group $G(\mathbb{F}_q)$ is governed by the structure of certain finite modular categories. I will describe some conjectures and results in this direction.

  14:35 – 15:30 **Maxim Nazarov**
  Title: Cherednik algebras and Zhelobenko operators.
  Abstract: This is a joint work with Sergey Khoroshkin. We study canonical intertwining operators between modules of the trigonometric Cherednik algebra, induced from the standard modules of the degenerate affine Hecke algebra. We show that these operators correspond to the Zhelobenko operators for the (untwisted) affine Lie algebra of type A. To establish the correspondence, we use the functor of Arakawa, Suzuki and Tsuchiya which maps certain modules of the latter algebra to modules of the Cherednik algebra.

All talks take place at the ESI, Boltzmann Lecture Hall!