Shape Design with Great Geometrical Deformations Using Continuously Moving Finite Element Nodes

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Abstract — **In this paper design sensitivity analysis is applied to solve the TEAM workshop problem 25. In order to justify the use of a gradient method, it is necessary to assume a continuously differentiable dependence of the stiffness matrix on the design parameters. Since design sensitivity analysis is mainly applicable to optimization problems, where the geometrical parameters undergo small changes only — which is not the case for the problem investigated in this paper — a procedure is proposed, which allows this method to be applied also when the changes in geometry are significant.**

Index terms —**Optimization methods, Sensitivity, Magnetostatics, Shape, Nonlinear Magnetics, Finite element methods**

I. Introduction

In order to avoid fake minima caused by abrupt changes in the finite element mesh topology [1], the nodes in the finite element mesh are varied continuously according to a mathematical relationship, describing the dependence of all movable nodes on the design parameters. This relationship leads to a faster computation than in case of a complete remeshing. Furthermore, from a theoretical point of view, knowing the functional dependencies of the moving nodes on the design parameters is a necessary requirement, if one uses smooth optimization methods taking advantage of analytically derived gradients. A method has been proposed, that allows such a continuous change of finite element nodes, when a non regular mesh is employed [2]. This method is applicable when the design parameters are not allowed to move within a wide range [3], since in this case the danger of distorting the Finite Element mesh too much becomes evident. An iterative procedure is proposed that remeshes the region of interest only when the distortion of the initial mesh would lead to inaccurate results.

II. TEAM Workshop Problem 25

Goal of the TEAM problem 25 [4] is to optimize the shape of die molds, used for producing anisotropic permanent magnets. The magnetic field is exited by a coil with a current density of 1.239219 A/mm^2. The shape of the die molds is parameterized by a circle for the inner (design parameter $p{=}R_1$) and an ellipse for the outer die ($p{=}L_2$ and $p{=}L_3$ respectively) (Fig. 1). The shape is to be set up in such a way that the magnetic flux density equals $0.35\cos\phi$ T for the x- and $0.35\sin\phi$ T for the y-component along the circle line A-B in 10 measurement points, at radius $r_0 = 0.01175\,m$.

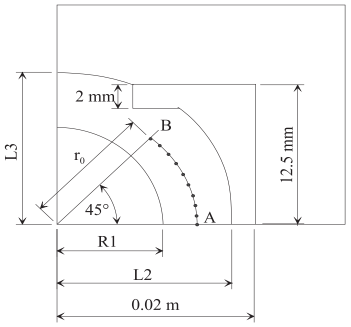

Fig. 1. TEAM 25-Region of Interest (exciting coil not shown here)

To fulfill the requirements a cost functional (1) was defined, where B_{xi} and B_{yi} are the calculated components of the magnetic flux density in the 10 measurement points, while B_{xi0} and B_{yi0} are the required values for these components.

$$\Psi = \sum_{i=1}^{10} \left(\left(B_{xi} - B_{xi0}\right)^2 + \left(B_{yi} - B_{yi0}\right)^2 \right) \qquad (1)$$

B_x and B_y are the x- and y-components of the magnetic flux density defined within a Finite Element as

$$B_x = \frac{1}{2\,\Gamma} \sum_{s=1}^{3} c_s\, A_s \quad \text{and} \quad B_y = -\frac{1}{2\,\Gamma} \sum_{s=1}^{3} b_s\, A_s, \qquad (2)$$

where

$$\begin{array}{ll} b_1 = y_2 - y_3 & c_1 = x_3 - x_2 \\ b_2 = y_3 - y_1 & c_2 = x_1 - x_3 \\ b_3 = y_1 - y_2 & c_3 = x_2 - x_1, \end{array}$$

Γ is the area of a Finite Element and A is the magnetic vector potential.

III. Mesh Generation

The elements used are linear triangular elements. Initially the finite element mesh is generated with Delaunay

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TABLE I
Starting values for the design parameters

R_{10} $[m]$	L_{20} $[m]$	L_{30} $[m]$
6.5E-3	15.3E-3	15.97E-3

triangulation (Fig. 3). The initial values for the design parameters resulting in a value of the cost functional of $\Psi = 2.0686E - 2$ are given in Table I. During the optimization process finite element nodes are changed in dependence on the design parameters. As long as these changes are not distorting the mesh too much (i.e. the angles of the elements are greater than a certain minimum angle), the nodes are shifted. Otherwise a new mesh is generated (Fig. 2). Additionally only the information concerning the nodes in the region of interest is updated in each iteration to save computation time [5].

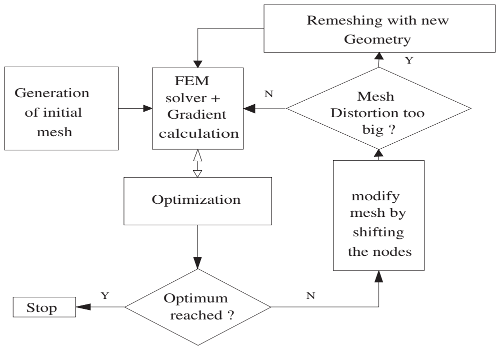

Fig. 2. Flowchart of the Procedure

In Fig. 4 the regions where the Finite Element nodes are moving are shown. The arrows indicate the direction in which the nodes are shifted. Within a region all node coordinates vary in a linear dependence on the design parameters. The abbreviations used in the following relationships for Ω_1 - Ω_9 can be obtained from Tables I, II, III and IV

- Ω_1: $\delta x = (x_{ell1} - x_{ini1})\,\frac{x}{x_{ini1}}$
- Ω_2: $\delta x = f_x\,\frac{(x-x_{ini2})[x_{ell1}-x_{ini1}-x_{ell2}+x_{ini2}]}{(x_{ini1}-x_{ini2})(x_{ell2}-x_{ini2})^{-1}}$
- Ω_3: $\delta x = (x_{ell2} - x_{ini2})\,\frac{x-0.02}{x_{ini2}-0.02}$
- Ω_4: $\delta r = (R_1 - R_{10})\,\frac{r}{R_{10}}$
- Ω_5: $\delta r = (R_1 - R_{10})\,\frac{0.01175-r}{0.01175-R_{10}}$
- Ω_6: $\delta x = (x_{ell3} - x_{ini3})\,\frac{x-\sqrt{r_{meas}^2-y^2}}{x_{ini3}-\sqrt{r_{meas}^2-y^2}}$
- Ω_7: $\delta x = (x_{ell3} - x_{ini3})\,\frac{x-0.02}{x_{ini3}-0.02}$
- Ω_8: $\delta x = (x_{ell3} - x_{ini3})\,\frac{(x_{ell1}-x_{ini1})-(x_{ell3}-x_{ini3})}{x_{ini1}-x_{ini3}}$
- Ω_9: $\delta r = (R_1 - R_{10})\,\frac{\sqrt{0.01^2+\frac{0.01}{\tan\phi}^2}-r}{\sqrt{0.01^2+\frac{0.01}{\tan\phi}^2}-R_0}$

Due to the special definition of the regions, where the Finite Element nodes are allowed to move, unacceptable distortions of the mesh leading to inaccurate results might occur. Then a remeshing with the new configuration has to be performed and the optimization procedure has to be restarted.

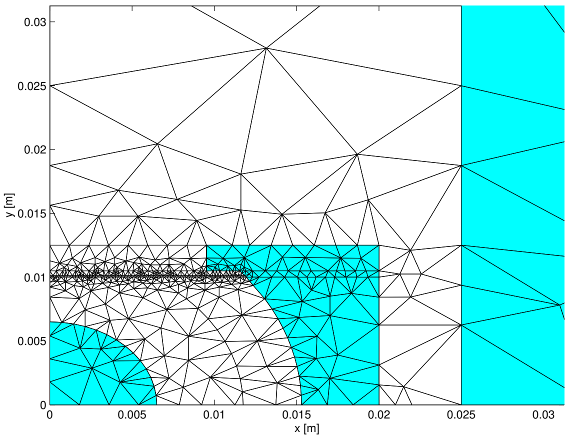

Fig. 3. Mesh generated with delaunay

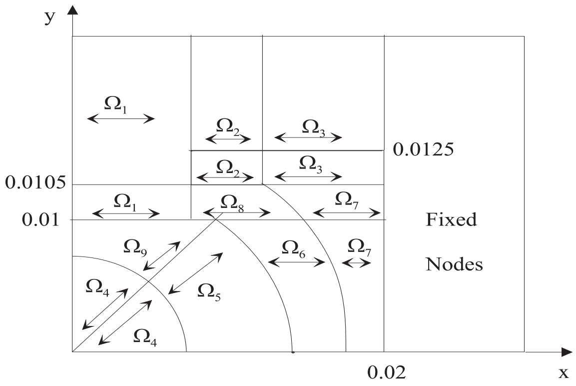

Fig. 4. Regions Ω_1 - Ω_9 of moving nodes

IV. Sensitivity Analysis

For the derivation of the stiffness matrix the following abbreviation is used:

$$g_{ij} = \frac{(b_i(p)\,b_j(p)\;+c_i(p)c_j(p))}{4\Gamma(p)}. \tag{3}$$

In order to compute the derivative of the cost functional with respect to the design parameters an adjoint equation is defined (4), that allows the computation of the gradient vector by solving only one linear system of equations.

For the left hand side of this system of equations the terms of the derivative of the stiffness matrix containing derivatives of the vector potential with respect to the design parameter p together with the corresponding stiffness matrix entries are assembled.

One entry of the stiffness matrix is depending on three vector potentials for one Finite Element, the derivatives of these multiplied with the vector potential A_s have to be written into the corresponding place in the adjoint matrix

TABLE II
Abbreviations 1

x_{ini1}	x_{ini2}	x_{ini3}
$L_{20}\sqrt{1-\left(\frac{0.0125}{L_{30}}\right)^2}$	$L_{20}\sqrt{1-\left(\frac{0.0105}{L_{30}}\right)^2}$	$L_{20}\sqrt{1-\left(\frac{y}{L_{30}}\right)^2}$

TABLE III
Abbreviations 2

x_{ini1}	x_{ini2}	x_{ini3}
$L_2\sqrt{1-\left(\frac{0.0125}{L_3}\right)^2}$	$L_2\sqrt{1-\left(\frac{0.0105}{L_3}\right)^2}$	$L_2\sqrt{1-\left(\frac{y}{L_3}\right)^2}$

TABLE IV
Abbreviations 3

r	ϕ
$\sqrt{x^2+y^2}$	$\frac{y}{x}$

in (4). This procedure leads to a symmetric matrix.

$$\sum_{j=1}^{n}\left(K_{ij}+\sum_{e}\left(g_{is}\frac{\partial\nu}{\partial B}\frac{\partial B}{\partial A_j}A_s\right)\right)\lambda_j=\frac{\partial\Psi}{\partial A_i} \tag{4}$$

In (4) $K_{ij}=\sum_e \nu^e\, g_{ij}^e$ (e denotes all Finite Elements which are shared by the current node), $1\leq i\leq n$ and ν is the magnetic reluctivity $\frac{H(B)}{B}$. The right hand side term in the equation system (4) is zero for all entries i that do not correspond to measurement nodes and for nodes where a homogeneous Dirichlet boundary condition is prescribed.

A. Derivative of the Cost Functional with respect to the Vector Potential

For the cost functional the flux density in finite element nodes is required. The flux density itself is computed as arithmetic mean of the flux densities of the elements that share the actual measurement node. Therefore the cost functional of (1) is rewritten to

$$\Psi=\sum_{i=1}^{10}\left(\left(\frac{1}{e}\sum_{j=1}^{e}B_{x\,i\,j}-B_{x\,i\,0}\right)^2+\left(\frac{1}{e}\sum_{j=1}^{e}B_{y\,i\,j}-B_{y\,i\,0}\right)^2\right). \tag{5}$$

Therefore the derivative of the cost functional with respect to the vector potential in the point l is given by

$$\frac{\partial\Psi}{\partial A_l}=\left\{\left(\frac{\sum_{j=1}^{e}B_{x\,l\,j}}{e}-B_{x\,l\,0}\right)\sum_{j=1}^{e}\frac{\partial B_{x\,l\,j}}{\partial A_l}+\left(\frac{\sum_{j=1}^{e}B_{y\,l\,j}}{e}-B_{y\,l\,0}\right)\sum_{j=1}^{e}\frac{\partial B_{y\,l\,j}}{\partial A_l}\right\}\frac{2}{e}. \tag{6}$$

Notice that the flux density depends on three vector potentials for each finite element and for the assemblation of the right hand side of the adjoint equation these derivatives have to be written into the corresponding locations of the right hand side vector.

B. Computation of the Gradient

For the optimization of the die press a direct dependence of the cost functional on the design parameters via the flux density (2) exists, therefore one entry of the gradient vector equals

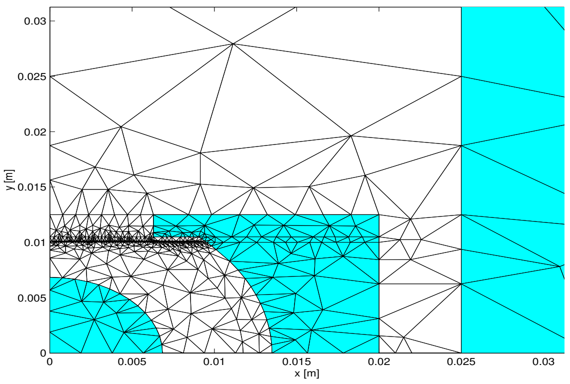

Fig. 5. Best configuration before remeshing

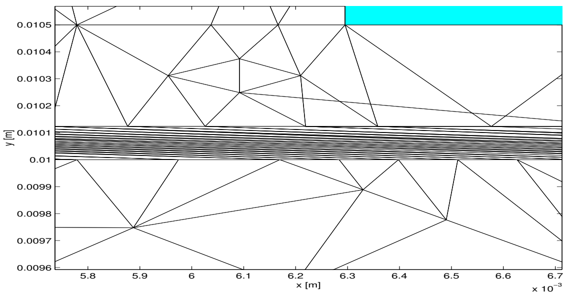

Fig. 6. Zoom in Fig. 5

$$\frac{d\,\Psi}{d\,p}=\frac{\partial\Psi}{\partial B}\frac{\partial B}{\partial p}-\lambda^T\left[\frac{\partial K}{\partial p}\right]_{A=const}A, \tag{7}$$

where λ is the solution of the system of equations (4). Again one has to take care of the special computation of B in the measurement points (mean value of the flux densities in the Finite Elements that share the same measurement node).

V. Solution of the Problem

The problem was solved by applying a Quasi-Newton method with a BFGS update for the Hessian [6].

In case when the mesh gets too much distorted during the optimization process, remeshing was performed and the optimization was restarted. This procedure was repeated until an optimal point was found and mesh distortion is acceptable.

During optimization the configuration in Table V is obtained. As it can be seen from Fig. 5 and Fig. 6 the initial mesh gets too much distorted to give accurate results and remeshing was necessary (Fig. 7).

In Table V the best configuration before remeshing resulting in a cost functional value of $1.273E-3$ are given. In Fig. 8 the required and calculated values for the x- and y-component of the magnetic flux density, resulting from this configuration, are shown. Point 1 is the measurement point where $\phi=0^o$ while point 10 is located in $\phi=45^o$.

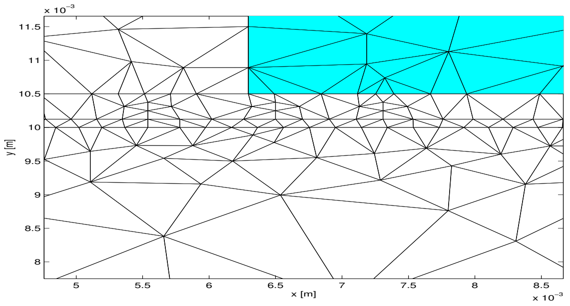

Fig. 7. Region in Fig. 6 after remeshing

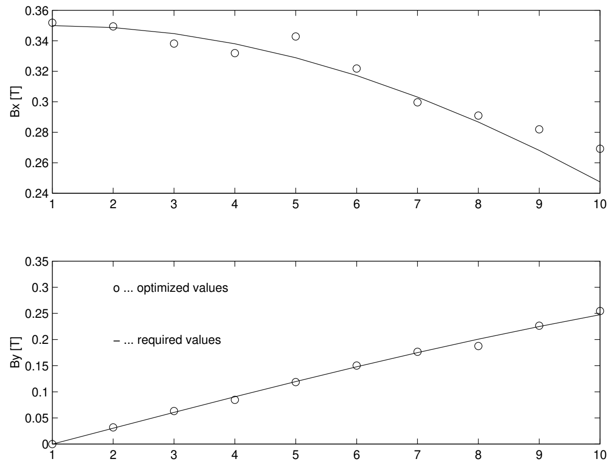

Fig. 8. B_x and B_y values after optimization with initial mesh

After remeshing the optimal point in Table VI with a cost functional value of $1.496E-3$ was found. This point leads to the B_x and B_y distributions in Fig. 9. Comparing Fig. 8 with Fig. 9 the better congruence of the calculated flux density and the required one in the points 8, 9 and 10, due to remeshing, is evident.

VI. Influence of Remeshing on the Cost Functional

To throw some light on the influence of remeshing on the cost functional value the flux densities in the measurement point at $\phi = 45^o$ according to (8) are calculated, because in the vicinity of this point mesh distortion is significant. For the configuration in Table V a function value of $\Psi\,|_{\phi=45^o} = 5.225E-4$ was obtained. After remeshing this value changes to $\Psi\,|_{\phi=45^o} = 1.95E-4$. And the complete cost functional value after remeshing equals $2.44E-3$.

$$\Psi \Big|_{\phi=45^o} = \left(B_{x10} - B_{x10\,0}\right)^2 + \left(B_{y10} - B_{y10\,0}\right)^2 \qquad (8)$$

TABLE V
Optimal Configuration before remeshing

R_1 $[m]$	L_2 $[m]$	L_3 $[m]$
6.9075E-3	13.566E-3	14.08E-3

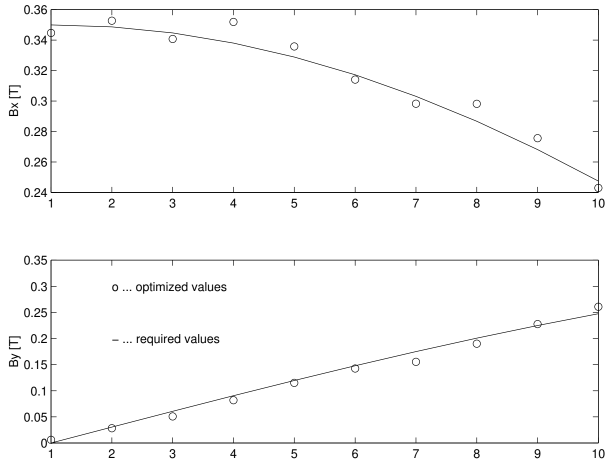

Fig. 9. B_x and B_y values with point in Table VI

TABLE VI
Optimal Configuration

R_1 $[m]$	L_2 $[m]$	L_3 $[m]$
7.04957E-3	13.47937E-3	14.15264E-3

VII. Conclusions

A method is proposed that allows the use of the sensitivity analysis, even when great geometrical deformations are possible. The discontinuities in the cost functional are not of danger for the optimization process, because a new optimization run is started after remeshing.

Although a suitable configuration can be easily achieved for the TEAM 25 problem, it is difficult to locate the optimal configuration where the *Kuhn-Tucker* conditions [6] are satisfied. The reason for this is a large value for the condition estimate for the projected Hessian [6].

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