Coarse space-discretization of bidomain equation: effect on conduction velocity and front shape

S. Pezzuto[a]

[a]Center for Computational Medicine in Cardiology
Università della Svizzera italiana
6900 Lugano, Switzerland
simone.pezzuto@usi.ch

The bidomain equation is the most commonly used model to describe in detail the spatial and temporal electric activity of the heart. The solutions are often travelling waves characterised by a very steep front, about 0.1 mm thick (in the longitudinal direction). This yields a considerable computational effort on patient-specific human geometries.

By means of a perturbation argument, it is possible to analyse in detail the effect of a specific discretisation in space on traveling wave solutions of the bistable (or Nagumo) equation, which is a simple but mathematically reasonable approximation of the dynamics at the front of the wave. We show how specific discretisation schemes (FD, FE, Hermite), and the choice of a coarse grid, can affect the solution, leading so to possibly erroneous physiological conclusions. We also analyse the impact of mass lumping, adopted by several authors in relation with operator splitting schemes.

Secondly, we exploit the error estimates to design a robust numerical scheme for the problem of interest. We propose and analyse two different schemes: the first one is a fourth-order (in space) scheme obtained by a weighted average of the finite difference and the finite element method. The second one is a "stabilised" finite element scheme, where we introduce a numerical conductivity to consistently adjust the conduction velocity.

Finally, we test the methodology on realistic ionic models and geometries. The results are in excellent agreement with our analytical results based on the bistable model, thus showing the validity of the approach.