Low Complexity Regularization of Inverse Problems: from Sensitivity Analysis to Algorithms

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Inverse problems and regularization theory is a central theme in imaging sciences, statistics and machine learning. The goal is to reconstruct an unknown vector from partial indirect, and possibly noisy, measurements of it. A now standard method for recovering the unknown vector is to solve a convex optimization problem that enforces some prior knowledge about its structure. This talk delivers some results in the field where the regularization prior promotes solutions conforming to some notion of simplicity/low-complexity. These priors encompass as popular examples sparsity and group sparsity, total variation and low-rank. Our aim is to provide a unified treatment of all these regularizations under a single umbrella, namely the theory of partial smoothness. This framework is very general and accommodates all low-complexity regularizers just mentioned, as well as many others. Partial smoothness turns out to be the canonical way to encode low-dimensional models that can be linear spaces or more general smooth manifolds. This review is intended to serve as a one stop shop toward the understanding of the theoretical properties of the so-regularized solutions. It covers a large spectrum including: (i) recovery guarantees and stability to noise, both in terms of ℓ_2-stability and model (manifold) identification; (ii) sensitivity analysis to perturbations of the parameters involved (in particular the observations); (iii) convergence properties of forward-backward type proximal splitting, that is particularly well suited to solve the corresponding large-scale regularized optimization problem.

This is a joint work with S. Vaiter, G. Peyré and J. Liang.