# On the Styrian Modelling Week

a.o.Univ.Prof. Mag.Dr. Stephen Keeling
Institute for Mathematics and Scientific Computing
Karl-Franzens University of Graz

Seventh Conference for Croatian Mathematics Teachers
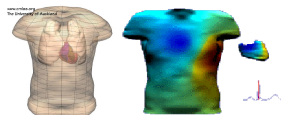June 29 – July 1, 2016
Zagreb, Croatia

# Overview

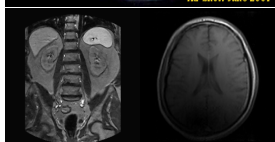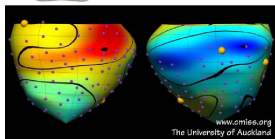**Introduction to the Styrian Modelling Week**

**Introduction to Modelling**

**Examples of Modelling Projects**

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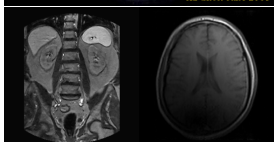- ▶ B.S. in biology and chemistry (with math and physics).

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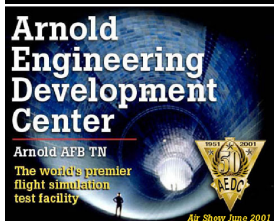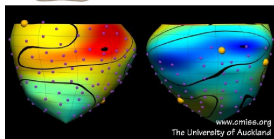- ▶ M.S. in biomedical engineering (bioelectric phenomena).

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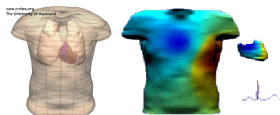- ▶ Ph.D. in mathematics (numerics for PDEs).

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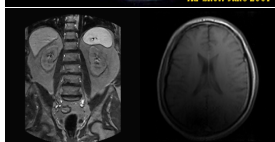- ▶ Residence in USA until 1998, in Graz, Austria since then, with American citizenship.
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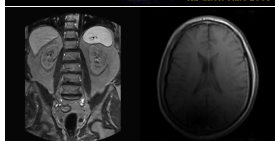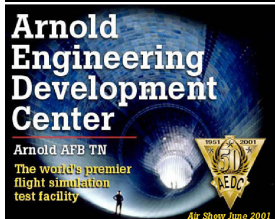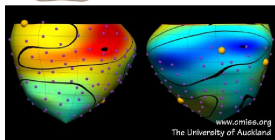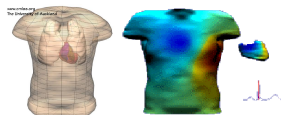


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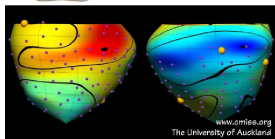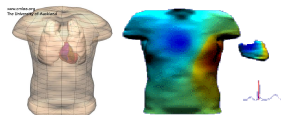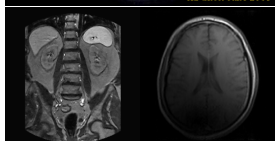- ▶ B.S. in biology and chemistry (with math and physics).
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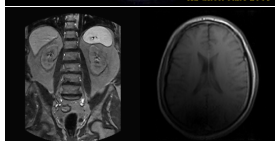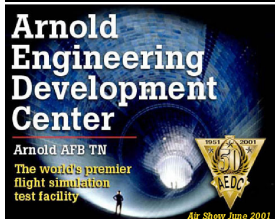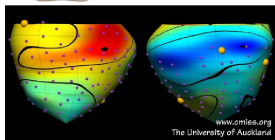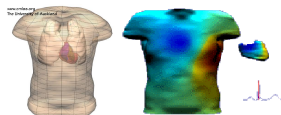


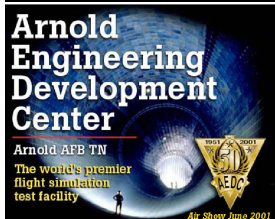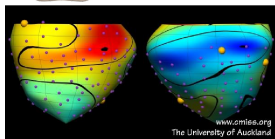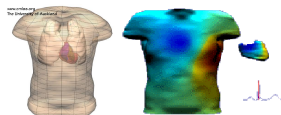- ► Residence in USA until 1998, in Graz, Austria since then, with American citizenship.
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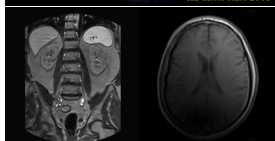- ► Habilitated in 2006 in Graz (mathematical image processing).

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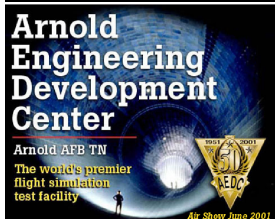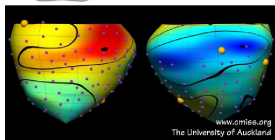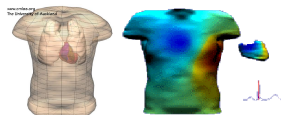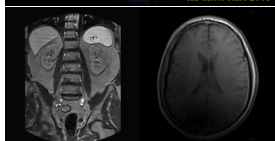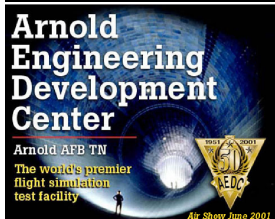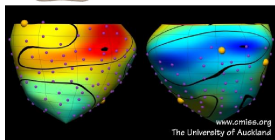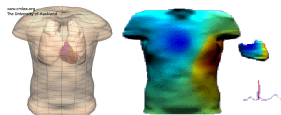

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- ▶ Organizer of the Styrian Modelling Week since 2005.

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- ▶ Participants are not expected to be acquainted with the project details beforehand.

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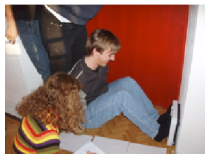


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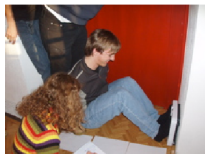


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- Daily schedule:
  - work together in the group outside 3 mealtimes, optional after dinner – often late at night!
  - unorganized free time (billiard, zoo) and organized free time activities (hiking, bowling, museums) etc.

# How is the Work Presented and Documented?



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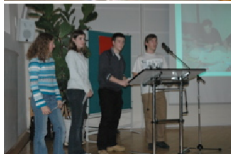


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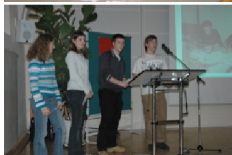


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- Persons invited to the presentations include:
    - parents and family,
    - teachers and school directors,
    - colleagues from neighboring countries,
    - leading members of the board of education,
    - leading members of the university as well as
    - the Austrian press.

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- Extensions of the project should be available.

# Previous Projects

2005:

- **Physiologie**: Herzkreislaufregulierung.
- **Biomechanik**: Bewegungen am Mond.
- **Mechanik**: Bewegung von Schüttgut in einer rotierenden Trommel.
- **Ökologie**: Nachhaltige Bewirtschaftung nachwachsender Ressourcen.
- **Sozialwissenschaft**: Wie können die demokratische Freiheit und der Reichtum eines Landes gleichmäßig verteilt werden?

2006:

- **Physiologie**: Gasaustausch in der Lunge.
- **Biomechanik**: Weitsprung mit Zusatzgewichten.
- **Mechanik**: Modellierung einer Seilbrücke.
- **Biowissenschaften/Sozialwissenschaften**: Ausbreitung ansteckender Krankheiten.
- **Bilderverarbeitung**: Interpolation von Bildern.

2007:

- **Physiologie**: Wie entfernt die Niere zuviel aufgenommenes Salz aus dem Körper?
- **Sportwissenschaften**: Sind Tore nur Zufall? Modellierung von Spielergebnissen bei Sportspielen
- **Neuroinformatik**: Künstliche neuronale Netze
- **Kryptographie**: "Feind hört mit!" – Geheimschriften und Verschlüsselung
- **Alternative Energie**: Lösungsansätze nach Installation eines fehlerhaften Erdwärmesystems

2008:

- **Sportwissenschaften**: Reflexion von Bällen
- **Geometrische Optimierung**: Verlegung einer Pipeline mit minimalen Kosten
- **Mechanik - Raumfahrt**: Weltraumaufzug
- **Kryptographie**: Ich bin ICH! - Bist Du wirklich DU?
- **Umwelt und Wirtschaft**: Produktionspitze des Erdöls

# Previous Projects

2009:

- **Medizin**: Der Bromsulphalein-Test
- **Sportwissenschaft**: Treffsicherheit im Sport
- **Geometrische Optik**: Verfolgung des Strahlenganges (ray tracing) in der geometrischen Optik
- **Spielstrategien**: Modellierung taktisch günstiger Laufwege beim Fußball
- **Forensische Wissenschaft**: Simulation des Einsturzes eines WTC-Turms

2010:

- **Physiologie**: Kompartimentanalyse für Hämodialyse
- **Hydrostatik**: Statische Schwimmlagen von Balken mit rechteckigem Querschnitt
- **Mechanik**: Domino Effekt
- **Ökologie**: Nachhaltige Forstwirtschaft
- **Bildverarbeitung**: Segmentierung von Bildern

2011:

- **Signalanalyse**: Geschwindigkeits- und Entfernungsmessung mit Signalwellen
- **Navigation**: Finden von optimalen Wegen in Umgebungen mit Hindernissen
- **Dynamische Systeme**: Gibt es Zeit-diskrete Pumpen?
- **Informationstechnik**: Googles PageRank-Algorithmus
- **Politikwissenschaft**: Entwicklung eines Wahlsystems

2012:

- **Sozialwissenschaften**: Modellierung des dynamischen Verhaltens von Fußgängern
- **Sport und Spiel**: Spielstärkeeinschätzung und Ergebnisvorhersage bei Paarvergleichen
- **Physik**: Die Weltraumflüge des Jules Verne
- **Informatik**: Bildverarbeitung mit neuronalen Netzen
- **Signalverarbeitung**: Trennung von Datenquellen in unkorrelierte und unabhängige Komponenten

# Previous Projects

2013:

- **Umweltpolitik**: Internationaler Klimaschutz
- **Sozioökonomische Physik**: Macht von Politikern und Fairness in einer Gesellschaft
- **Wirtschaftswissenschaften**: Financial Contagion
- **Wärmetransport**: Entwicklung einer Fußbodenheizung
- **Kunst und Fotografie**: Erstellung und Beurteilung eines Fotomosaiks

2014:

- **Akustik**: Musikinstrumente
- **Ingenieurtechnik**: Optimale Steuerung einer Heizung
- **Optimale Steuerung**: Optimale Steuerung eines Chaser-Satelliten zur Weltraumschrottbeseitigung
- **Digitale Bildverarbeitung**: Steganographie und Bilder
- **Medizin**: Voraussage des Ergebnisses einer Cerclage Operation

2015:

- **Inverse Probleme**: Shape From Shading - Vom Schattenbild zur Oberfläche
- **Ingenieurtechnik**: Wie(so) funktioniert Segway fahren?
- **Computergrafik**: Voraussage einer Beobachtung durch Strahlenverfolgung
- **Dynamische Systeme**: Chaos und fraktake Geometrie
- **Sozialwissenschaften**: Optimierung des Verkehrsflusses

2016:

- **Himmelsmechanik**: Lagrange-Punkte im Sonnensystem
- **Informatik**: Wie funktionieren Empfehlungsdienste?
- **Straßenverkehr**: Voraussage des Verkehrs in einem Transportnetzwerk
- **Finanz**: Entwicklung von Tradingstrategien
- **Kontinuumsmechanik**: Deformationswellen elastischer Materialien

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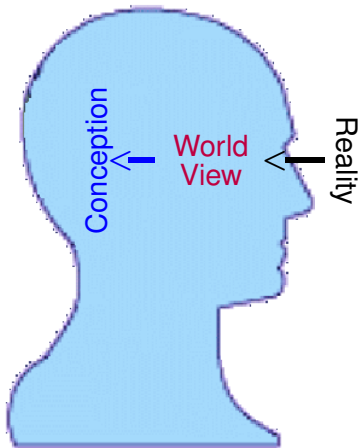- At the Styrian Modelling Week it is often asked by novices:

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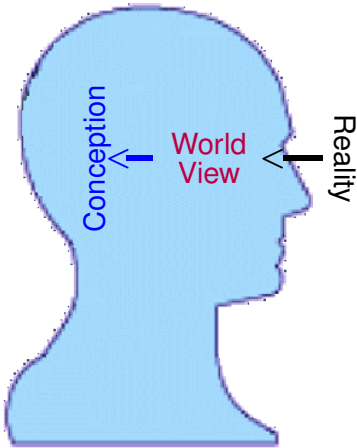  - But this does not fit to the context.
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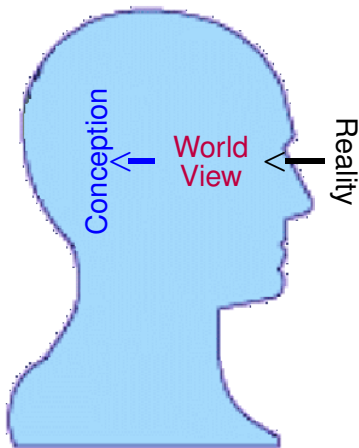# What is Modelling?

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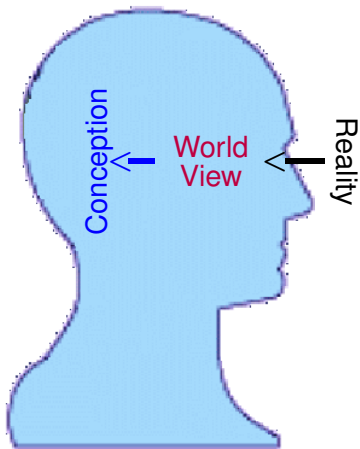  - ▶ A new-born child gets flooded with perceptions.

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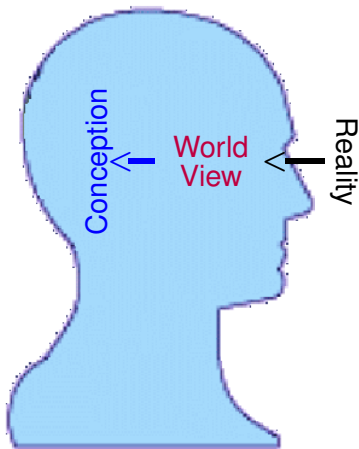
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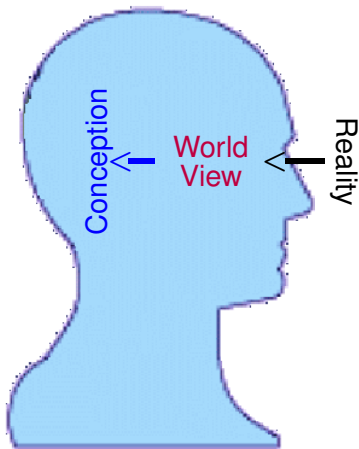
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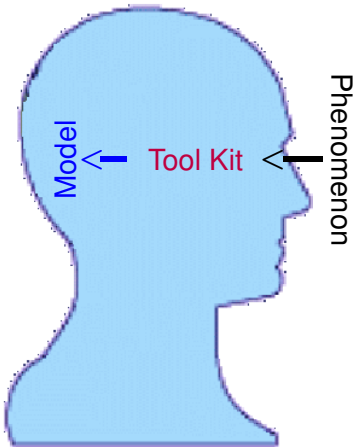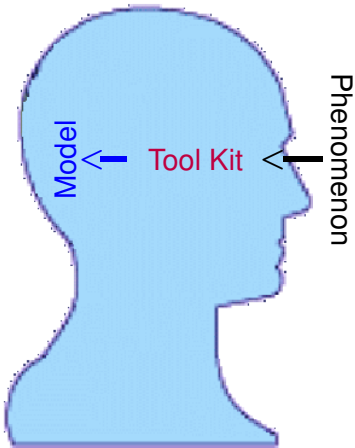# What is Modelling?

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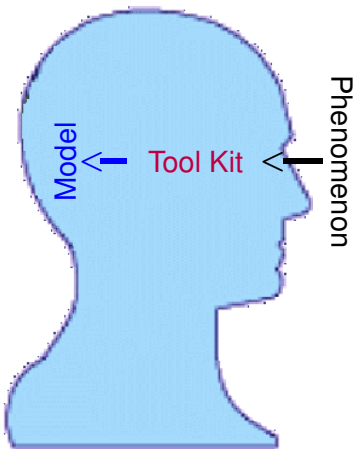
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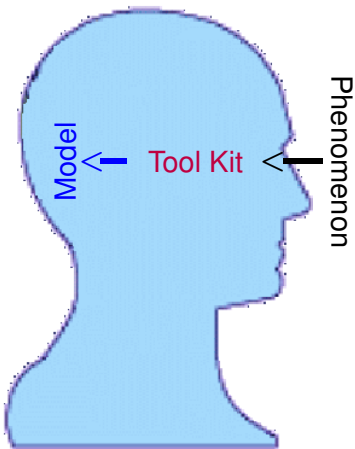
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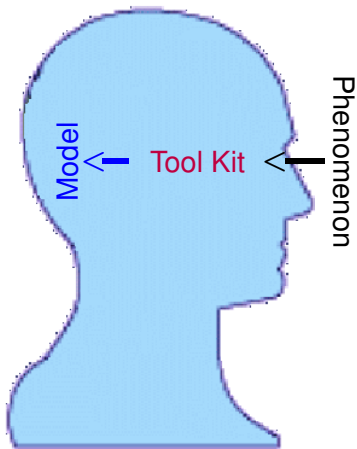# What is Modelling?
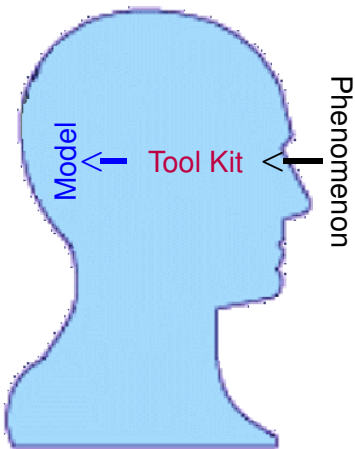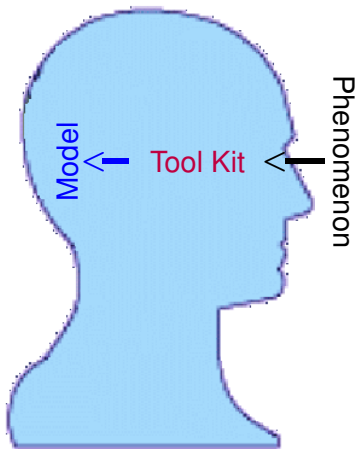
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Model ← Tool Kit ← Phenomenon

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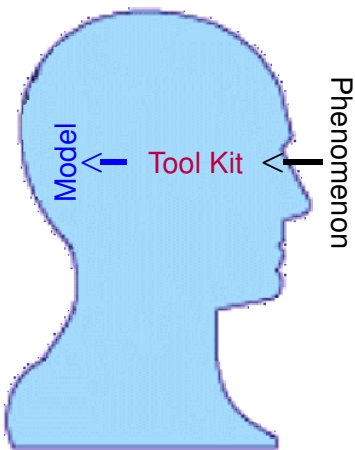
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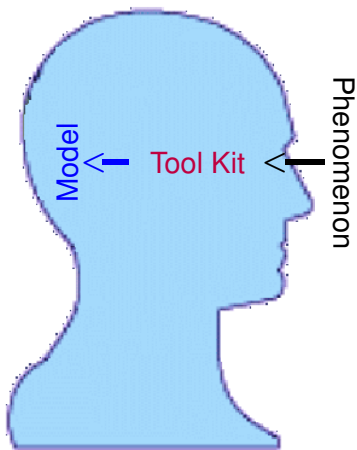
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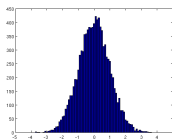$$E = mc^2 \quad \text{or} \quad F = ma \quad ?$$

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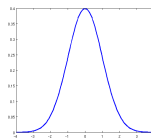- ▶ Example: Samples of one's own weight measured daily and represented with a histogram (left):
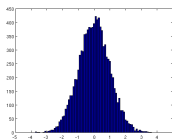
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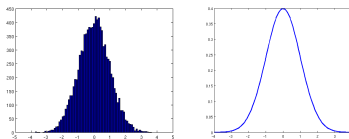  Is the deeper *reality* a smooth approximated curve (right)? What are probabilities?
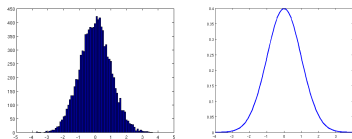
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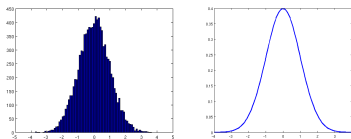- ▶ Main point: These descriptions are only provisional *models*, which have limits to their validity.

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For the documentary film *Supersize Me!*

Recall that one simplifies and makes assumptions depending upon goals.

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- consumes about 5000 kcal/day,
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Step 1: Definition of a physical domain,

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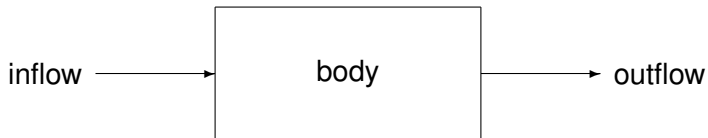
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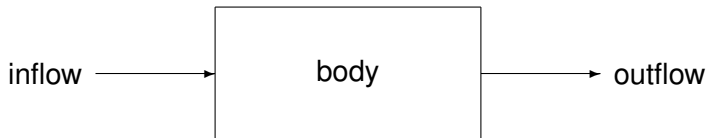


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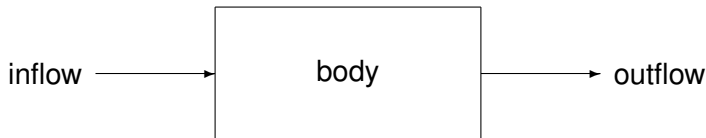
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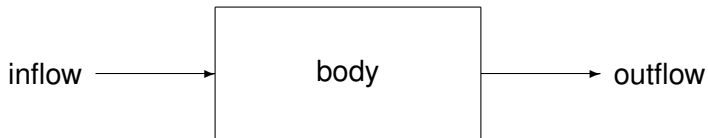
Step 1: Definition of a physical domain,
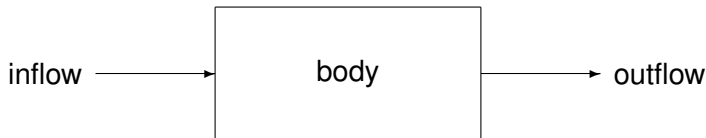


Collection of assumptions,

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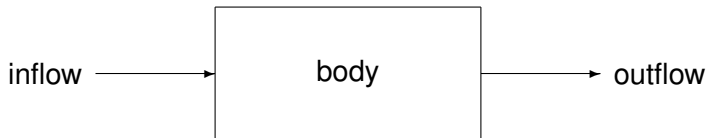# Step 1 of Modeling
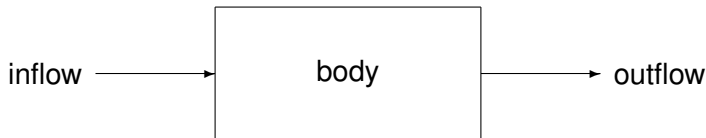
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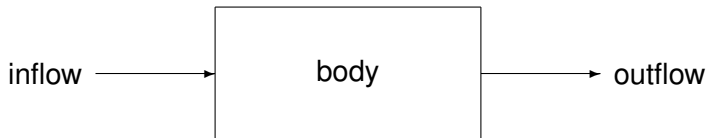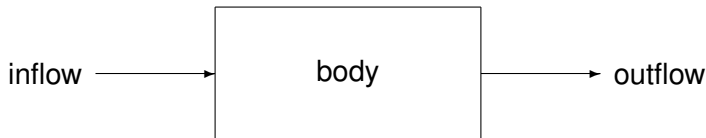
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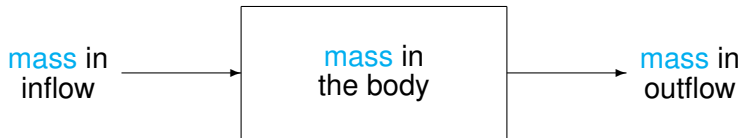
Step 2: Symbolic description of the system,

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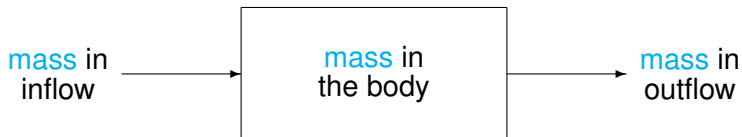
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Identification of an applicable principle: mass balance
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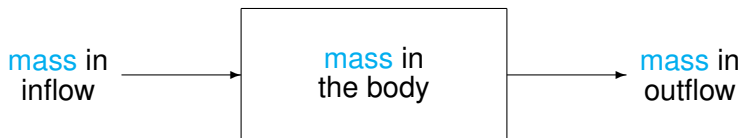


Identification of an applicable principle: mass balance
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$$\text{mass change in the body (1 day)} = \text{mass inflow (1 day)} - \text{mass outflow (1 day)}$$

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Mathematical formulation of the principle:

# Step 2 of Modellierung

$$\begin{array}{ccc}
\text{mass change in} & = & \text{mass inflow} & - & \text{mass outflow} \\
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\end{array}$$

Mathematical formulation of the principle:

$$m(1) - m(0) =$$

where

$m(1)$ = mass at the end of the 1. day
$m(0)$ = mass at the beginning (84kg)

# Step 2 of Modellierung

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| | | |
|---|---|---|
| $m(1)$ | $=$ | mass at the end of the 1. day |
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| $z(0), a(0)$ | $=$ | inflow, outflow for the forthcoming 1. day |

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and similarly for the subsequent days.

# Step 2 of Modellierung

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Symbolic answers to the goal-questions:

$$m(30) =? \quad m(\infty) =?$$

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Detailed description of the system: inflow?

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- ▶ density for a typical McDonalds mixture:
$$7.8\frac{kcal}{g} = 7800\frac{kcal}{kg}$$

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- density for a typical McDonalds mixture:
$$7.8\frac{kcal}{g} = 7800\frac{kcal}{kg}$$
- mass inflow: energy inflow / density
$$5000\frac{kcal}{day} \bigg/ 7800\frac{kcal}{kg} = \frac{5000}{7800} \frac{kg}{day}$$

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- In particular for the forthcoming 1. day,

$$a(0) = \frac{21.6}{7800} \times m(0)$$

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- In particular for the forthcoming 1. day,
  $$a(0) = \frac{21.6}{7800} \times m(0) = \frac{21.6}{7800} \times 84$$

Step 3: Solution of the mathematical problem – *numerically*!

# Step 3 of Modelling

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- Summary for the 1. day,

$$m(1) = m(0) + z(0) - a(0) = 84 + \frac{5000}{7800} - \frac{21.6}{7800} \times 84 = 84.8736$$

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$$m(2) = m(1) + z(1) - a(1) = 84.8736 + \frac{5000}{7800} - \frac{21.6}{7800} \times 84.8736$$
$$= 85.7497$$

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- etc, until the 30. day,

  $$m(30) \approx 95.8 \text{kg}.$$

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or

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# Step 3 of Modelling

Step 3: Solution of the mathematical problem – *exactly*!

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▶ The mass balance,

$$m(t+1) - m(t) = z(t) - a(t)$$

for 1 day at the beginning of the $t$-th day.

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- With $\Delta t \to 0$,

$$\underbrace{m'(t)}_{\substack{\text{instantaneous} \\ \text{rate of change}}} \xleftarrow{\Delta t \to 0} \frac{m(t + \Delta t) - m(t)}{\Delta t} = z(t) - a(t)$$

  cf. instantaneous and average speed.

# Step 3 of Modelling

- With the inflow $z(t) = \epsilon/\kappa$ ($\epsilon = 5000$, $\kappa = 7800$) and outflow $a(t) = m(t)\phi/\kappa$ ($\phi = 21.6$) there results the *differential equation*,

$$m'(t) = \epsilon/\kappa - m(t)\phi/\kappa \quad \begin{cases} > 0, & m(t) < \epsilon/\phi \ (\approx 231.5) \\ < 0, & m(t) > \epsilon/\phi \ (\approx 231.5) \end{cases}$$

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- Solutions mit $m(0) = 84$ and $m(0) = 400$,

Step 4: Qualitative investigation of the mathematical model

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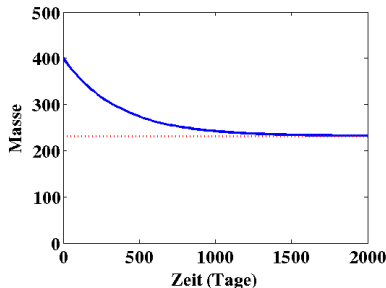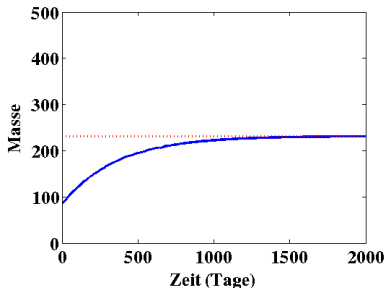
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- Do results depend upon the initial weight $m(0)$?

# Step 5 of Modelling

Step 5: Comparison with data, validation

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- The computed value is $m(30) = 95.8$ kg.

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  - ▶ If departures from the predicted sequence were not randomly scattered but were instead systematic?
- ▶ If differences between measured and predicated values are significant, which changes in the model should be considered next?

# Modelling Project – 2005

**Social Science: How Can the Democratic Freedom and the Wealth of a Country be Uniformly Distributed?**

▶ The toolkit: multi-agent simulation with Netlogo

▶ social behavior of interacting individuals

*Sugarscape: Distribution of Wealth*
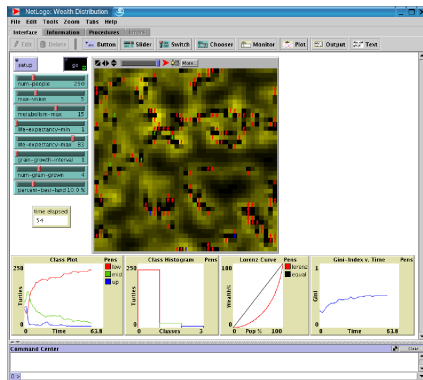
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► Two modelling perspectives:

pressure, temperature, density
(macroscopic properties)

*top down*
(description)

*bottom up*
(emergence)

collisions between molecules,
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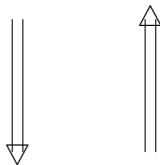
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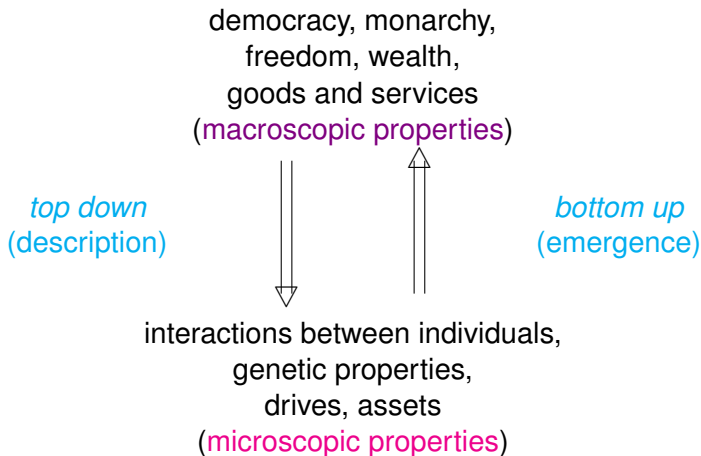
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drives, assets
(microscopic properties)
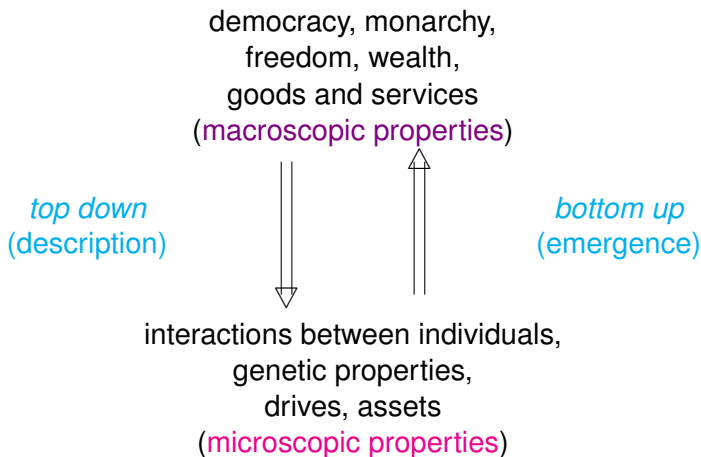
▶ Bottom up modelling was used for this project to let an economy emerge with distributed freedom and wealth.

# Modelling Project – 2006

## Image Processing: Interpolation of Images

- ► How to fill in the details of the motion?



$I_0$             $I_1$

$I_0$             $I_1$

# Modelling Project – 2006

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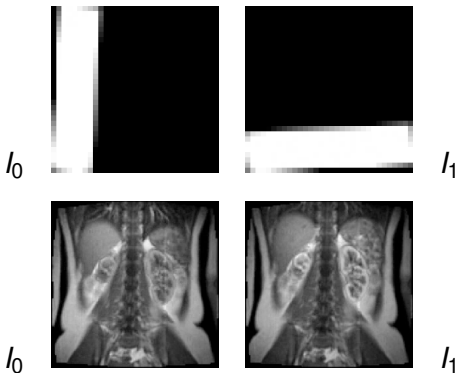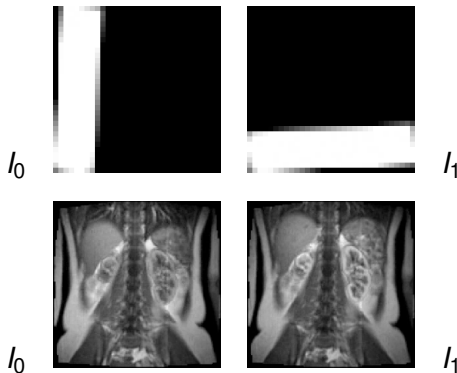
► Interpolation with $I(t, x) = (1 - t)I_0(x) + tI_1(x)$ ?
Gives only a fade-in / fade-out !

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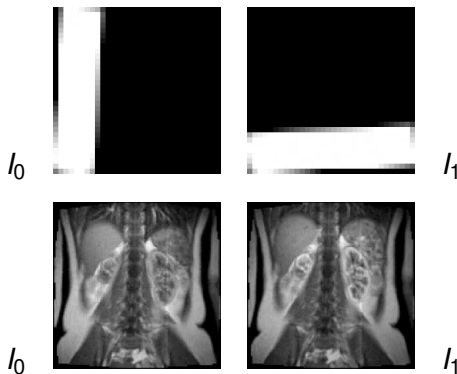
- ► Interpolation with $I(t, x) = (1 - t)I_0(x) + tI_1(x)$ ?
  Gives only a fade-in / fade-out !
- ► One computes a transformation $y(t, x)$ with $y(0, x) = x$ and
  $I_0(y(1, x)) \approx I_1(x)$ taking $I(t, x) = I_0(y(t, x))$. [result]

# Modelling Project – 2007

## Alternative Energy: Solution Approaches after Installation of a Faulty Heat Pump System

- ▶ A faulty heat pump system had been installed.
- ▶ The consumer must make an informed decision.
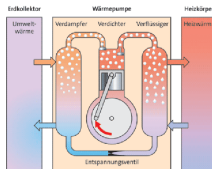
# Modelling Project – 2007

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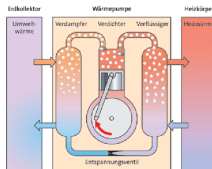


- ► All information from the building company and from the heat pump company was made available.

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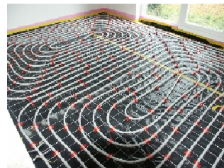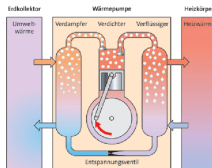


- ▶ All information from the building company and from the heat pump company was made available.
- ▶ The project was to perform simulations and to make an informed decision for the consumer.

# Modelling Project – 2008

## Environment and Economy: Peak Oil Production

▶ A collision between supply and demand:



$P_{\max}$, $P(t)$
$P_{\min}(t)$
fluctuations
in price
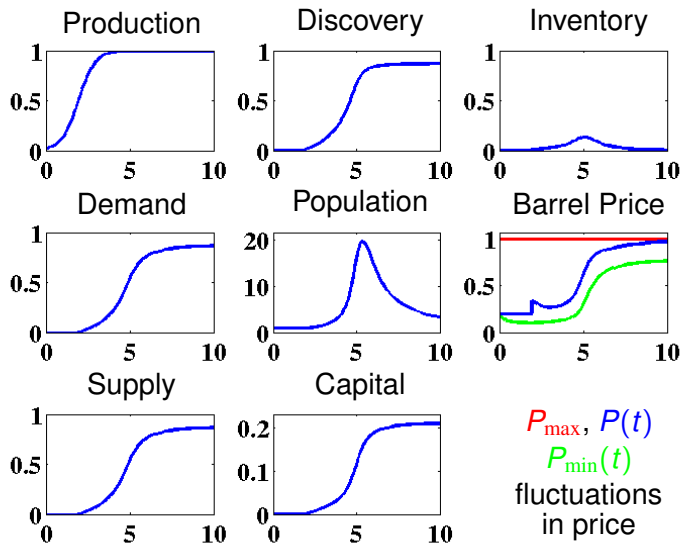
# Modelling Project – 2009

**Forensic Science: Simulation of the Collapse of the WTC Towers**

Project Result:

Netlogo Code:      `wtc.nlogo`

also in MATLAB:     `wtc.m`
and in EXCEL:      `wtc.xls`

Webpage:
```
http://imsc.uni-graz.at/
        keeling/wtc.html
```

Description:      `wtc1.html`

Derivation:      `wtc2.html`
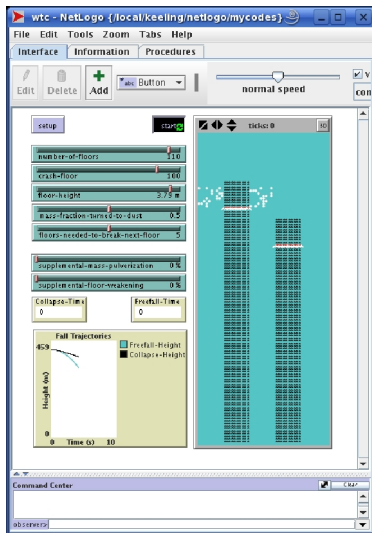
**Image Processing:** Segmentation of Images

► How to identify all points belonging to a single object?
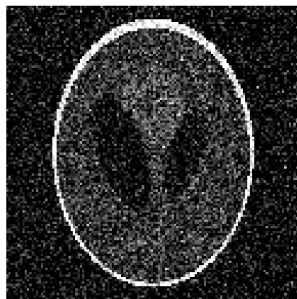
# Modelling Project – 2010

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- ► How to identify all points belonging to a single object?
- ► Easy if intensities are piecewise constant.
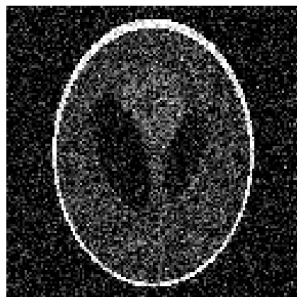
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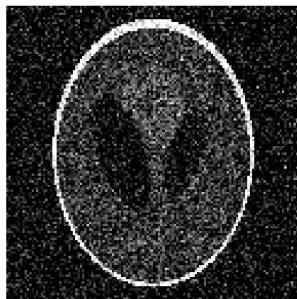


- Otherwise, ($K$-means) find average intensities $\{\mu_k\}_{k=1}^{K}$,

$$\min_{\mu_k} \sum_{i \in I_1} |\mu_1 - x_i|^2 + \cdots + \sum_{i \in I_K} |\mu_K - x_i|^2$$

over segments $I_k = \{i : |x_i - \mu_k| < |x_i - \mu_l|, l \neq k\}$.

# Modelling Project – 2011

**Political Science:** Development of a Voting System

Example: beauty contest



A    B    C

# Modelling Project – 2011

**Political Science:** Development of a Voting System

Example: beauty contest



A      B    C

- ▶ 3 judges: CBA, 2 judges: BAC

# Modelling Project – 2011

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Arrow's (Impossibility) Theorem: Under a list of *reasonable* conditions for fairness, there exists no rule to establish a fair preference list for a group.

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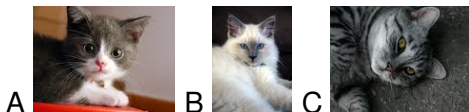
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Arrow's (Impossibility) Theorem: Under a list of *reasonable* conditions for fairness, there exists no rule to establish a fair preference list for a group.

- Yet Arrow's Theorem can be circumvented by *fuzzy* votes, i.e., a weighted distribution of preferences.

# Modelling Project – 2012

## Signal Processing: Separation of Data Sources into Uncorrelated and Independent Components

Motivating Example: The cocktail party problem
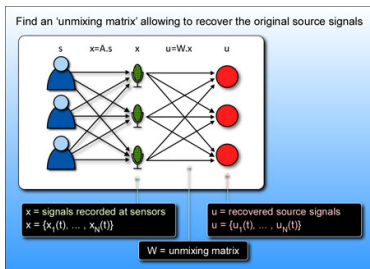
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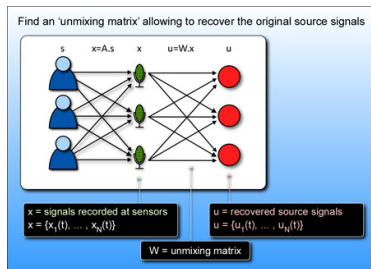


- Statistically independent sources $s(t)$ are unknown.

# Modelling Project – 2012

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Motivating Example: The cocktail party problem
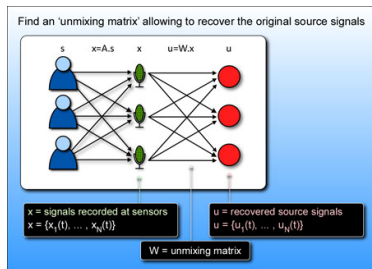


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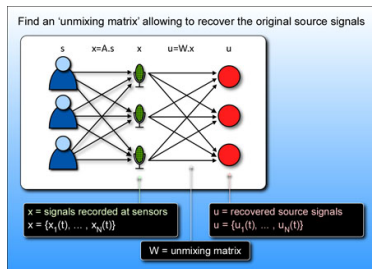


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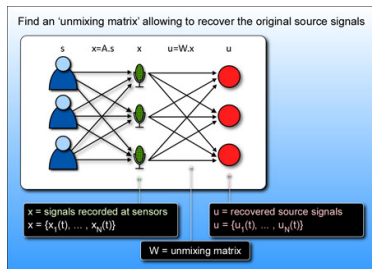


- ▶ Statistically independent sources $s(t)$ are unknown.
- ▶ These are measured in different mixtures $x(t) = As(t)$.
- ▶ The mixing matrix $A$ is also unknown!
- ▶ The task can be performed approximately with an eigen-space decomposition of the correlation matrix. [before] [after]

# Modelling Project – 2013

## Art and Photography: Creation and Evaluation of a Photo Mosaic

- Given: a target photo and many single photos.

# Modelling Project – 2013

**Art and Photography: Creation and Evaluation of a Photo Mosaic**

- Given: a target photo and many single photos.



- A photo mosaic should be created from the single photos, which is somehow similar to the target photo.

# Modelling Project – 2013

## Art and Photography: Creation and Evaluation of a Photo Mosaic

- Given: a target photo and many single photos.



- A photo mosaic should be created from the single photos, which is somehow similar to the target photo.
- The difficulties lie in the details:
    - What is a *match* between target cell and single photo?
    - A target cell may match well to many single photos.
    - A single photo may match well to many target cells.
    - How to perform the optimization efficiently?

**Medicine:** **Prediction of the Result of a Cerclage Operation**

▶ To treat retinal detachment or tear
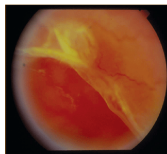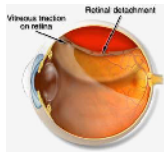


a rubber band (cerclage) is attached to the eyeball.

# Modelling Project – 2014

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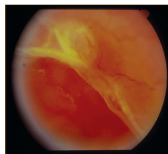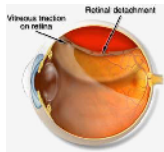


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► How to predict the intra- and post-operative states
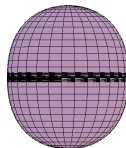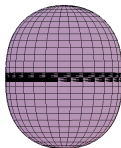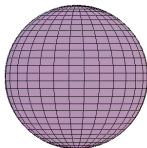


after the pressure and volume have normalized?

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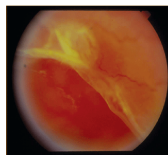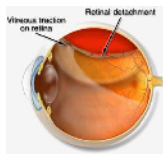


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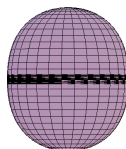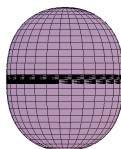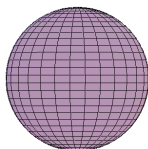


after the pressure and volume have normalized?

▶ One must establish a pressure-volume relation and perform a force balance to find the geometry.

# Modelling Project – 2015
## Social Sciences: Optimization of Traffic Flow



jam without accident
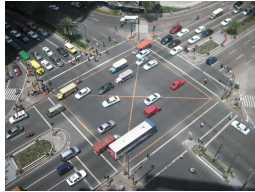


asynchronous city flow

# Modelling Project – 2015
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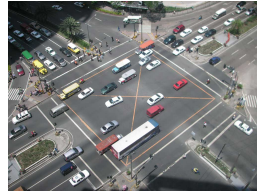


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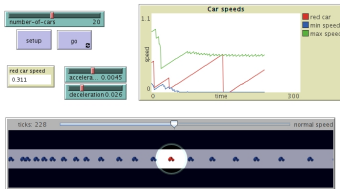


Netlogo Traffic Basic
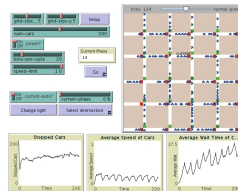


Netlogo Traffic Grid

# Modelling Project – 2015
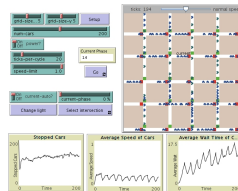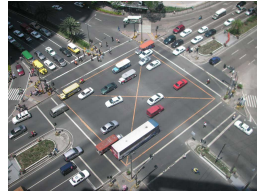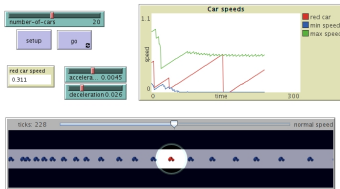## Social Sciences: Optimization of Traffic Flow



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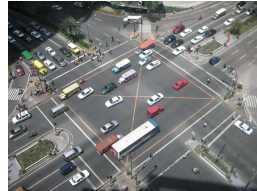


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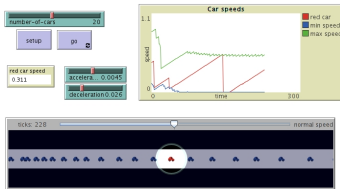


Netlogo Traffic Basic



Netlogo Traffic Grid

► How to maximize the flow?

# Modelling Project – 2015
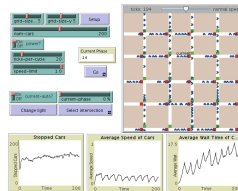## Social Sciences: Optimization of Traffic Flow



jam without accident



asynchronous city flow



Netlogo Traffic Basic



Netlogo Traffic Grid

- ▶ How to maximize the flow?
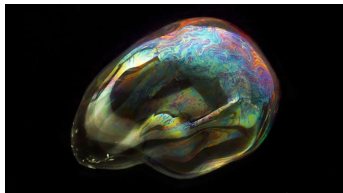- ▶ One develops relations among density, velocity and flow.

# Modelling Project – 2016

## Continuum Mechanics: Deformation Waves of Elastic Materials

▶ How does one model such large deformations?



bungee jumping



soap bubbles
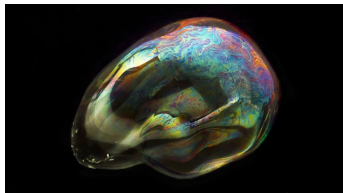
# Modelling Project – 2016

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- In general one uses Lagrangian mechanics with a non-linear potential energy.

# Modelling Project – 2016

## Continuum Mechanics: Deformation Waves of Elastic Materials

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bungee jumping           soap bubbles

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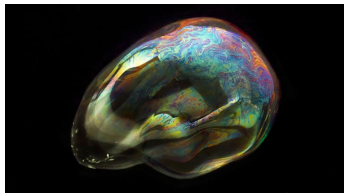
► A simplified approach involves a network of masses connected by springs!

# Thank You for the Invitation!

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**Thank You for Your Attention!**