START PROJECT: INTERFACES AND FREE BOUNDARIES

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START Project, University of Graz

i/fb START PROJECT: INTERFACES/FREE BOUNDARIES

START Project

Member

- Head of Project: Michael Hintermüller
- Post Doc: Yiqiu Dong, Antoine Laurain, Hicham Tber
- PhD Student: Monserrat Rincon, Ian Kopacka (former)
- Technical Assistant: Martin Kanitsar

Research Area

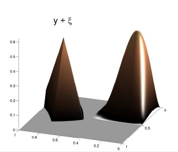
- Mumford-Shah based segmentation
- State-dependent regularization
- Multi-model registration and segmentation
- Domain decomposition in medical imaging
- Mixed control-state constrained problems
- Coupled complementarity problems in Biomed
- Nonsmooth parameter identification

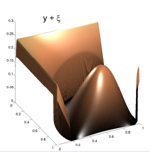
Mathematical Programs with Equilibrium Constraints (MPEC)

- MPEC: Optimal control of a variational inequality / complementarity system
- Difficulties: complementarity system violates constraint qualifications $\Rightarrow$ cannot apply standard optimization theory
- Application: Optimal control of elasto-plastic torsion; obstacle problem; Stefan problem; pricing of American options; etc.

Mathematical Programs with Equilibrium Constraints (MPEC)

- Introduction of appropriate stationarity concepts
- Design of algorithms with stable behavior under mesh refinement
- Semismooth Newton solvers and nonlinear multigrid
- Example: optimal control of obstacle problem





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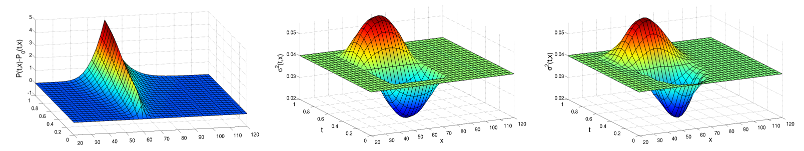
- Example: Local volatility identification in American option pricing
- First order optimality condition of C-stationarity-type.
- Active-set equality constrained Newton solver with feasibility restoration.



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Cahn-Hilliard Equations

- Model describes the process of phase separation (solidification and melting process, multi-phase flow, crack propagation, ...).
- Semi-implicit scheme for time.
- Formulation of the semi-discrete problem as a mathematical program in Banach space.
- Path-following combined with semi-smooth Newton method to solve the mathematical program.

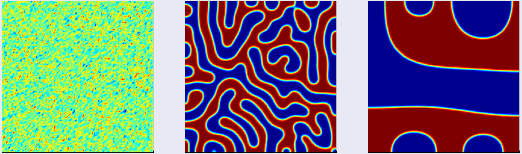

Figure: u at time t=0 s, t=0.1 s and t=0.5 s

- Goal-oriented approach: control, mixed control-state or control and state constrained optimal control of partial differential equations.
- Residual approach: elliptic control problems with control constraints, Cahn-Hilliard model with double obstacle free energy, ...

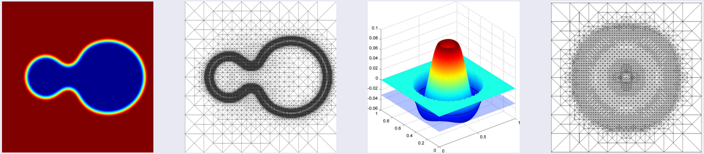

Figure: Residual approach (left): order parameter and corresponding mesh at time t=0.005 s.
Goal-oriented approach (right): desired state and refined mesh for elliptic PDE with state and control constraints.

Electrochemical Machining Problem

- Problem setting: Shape a metal part by placing it as an anode in a electrolytic cell.
- Issue: The problem is modeled by a variational inequality (VI). Control the active region with the shape of the domain.
- Issue: The shape gradient is non-linear.
- Our approach: Regularize the VI, use the shape derivative for the regularized problem

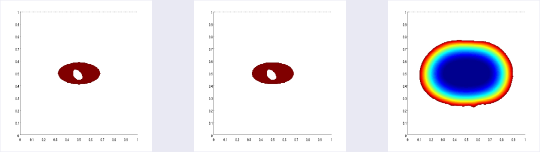

Figure: Target (left), active set (center), Solution (right)

State-Constrained Problem

- Problem setting: Solve the obstacle problem by considering it as a free boundary problem.
- Our approach: We use the tools of shape optimization and a level set method to solve the problem
- Motivation: Mesh dependence of existing algorithms. Algorithm in the infinite-dimensional setting.

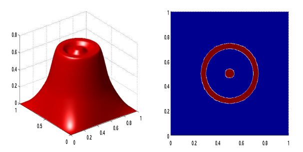

Figure: Solution u (left) and active set (right)

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h	AS iterations	Topology iterations	Shape iterations
$1/128$	18	2	2
$1/256$	31	4	2
$1/512$	59	3	4

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Image Segmentation

- Problem setting: Segment an image into separate parts using the Mumford
- Model: Minimize Mumford-Shah functional
- Our approach [Hintermüller, Laurain]: We are interested in piecewise constant models using a level set representation of the regions.
- Use Topological and Shape derivatives.
- Very fast algorithm...

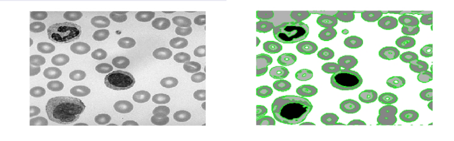

Figure: Original image (left), segmentation with contour in green (right)

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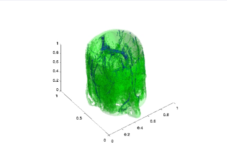
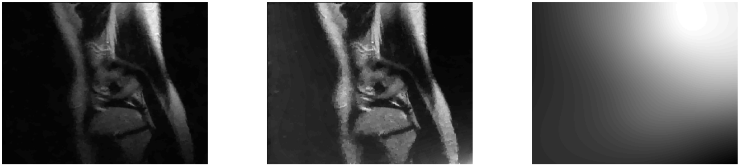


Figure: $256 \times 208 \times 70$ image. The algorithm converged after 30 iterations in 4.93s on a standard desktop PC.

Modulation recovery in MRI

- Problem setting: Recover unknown modulations in images due to surface coil acquisition in MRI using the piecewise constant segmentation, without a priori knowledge on the position of the coil.
- Model: Minimize a generalized Mumford-Shah functional which includes the modulation as an unknown.
- Use Topological and Shape derivatives.

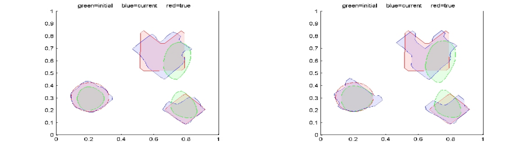


Original image (left), Reconstruction (center), Modulation (right)

Electrical Impedance Tomography (EIT)

- Problem setting: Find electrical properties in the interior of the body, such as conductivity, from measurements of electric currents f / voltages v on Σ.
- Our approach [Hintermüller, Kanitsar, Laurain]: We are interested in piecewise constant models and reconstructions using topological and shape derivatives .
- Issue: Inverse problem severely ill-posed!

Reconstructions for 1% (left), 3% (right) noise.



red ... original; blue ... reconstruction; green ... initialization.

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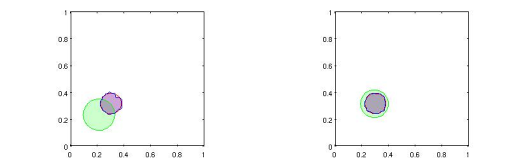
Higher-order topological expansion for EIT

- Is it possible to improve the initialization given by the topological derivative by using higher-order topological expansion ? We computed these higher-order expansions for the EIT problem.
- Provide a better initialization of the problem.
- Non local terms appear in the topological derivative.



red ... original; blue ... reconstruction; green ... initialization.
After 300 iterations using first-order expansions (left), after 50 iterations using second-order expansions (right)

Spatially Dependent Parameter Selection in Multiscale Total Variation

- Problem: Select the regularization parameter in multiscale TV model for blurred noisy images restoration.
- Our approach: Based on the scales of the features, we propose the local variance estimator to select the parameter adaptively.
- Based on Fenchel-dulity and inexact semismooth Newton techniques, we propose a superlinearly convergent algorithm.
- The whole algorithm is parameter-free.

Noisy image (left), TV result (center), Our result (right)

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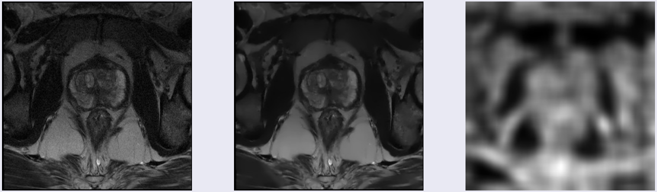
Spatially Dependent Parameter Selection in Multiscale Total Variation

- Restoration of MRI



Noisy image (left), Our result (center), Parameter value (right)

- Restoration of Blurred Noisy Color Image

Blurred noisy image (left), Our result (center), Parameter value (right)

Primal-Dual Method for L1-TV Image restoration

- Problem: Solve L1-TV Model in order to deblur and remove impulse noise.
- Issue: L1 data-fidelity term is nonsmooth, and TV regularization term is nondifferentiable.
- Our approach: Based on Fenchel-dulity and inexact semismooth Newton techniques, we propose an efficient primal-dual algorithm.
- Combined with the noise detectors, our method is improved on the capability of noise removal.

Blurred noisy image with low noise level (left), FTVd result (center), Our result without noise detector (right)

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- Combined with the noise detectors, our method is improved on the capability of noise removal.

Incomplete blurred image (left), Incomplete blurred noisy image (center), Our result (right)

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Primal-Dual Method for L1-TV Image restoration

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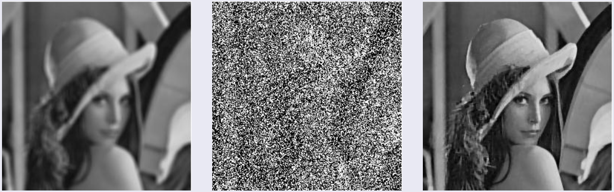

Blurred image (left), Blurred noisy image with 70% impulse noise (center), Our result with noise detector (right)

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Relevant Publications:

1. M. Hintermüller and A. Laurain, *A shape and topology optimization technique for solving a class of linear complementarity problems in function space*. IFB-Report No. 10 (10/2007). Published in: Computational Optimization and Applications, 2008.
2. M. Hintermüller and A. Laurain, *Electrical Impedance Tomography: From Topology to Shape*. IFB-Report No. 15 (02/2008). Published in: Control and Cybernetics, special issue on the occasion of Jean-Paul Zolésio's 60th birthday, Vol. 37, No. 4, 2008.
3. M. Hintermüller and A. Laurain, *Multiphase image segmentation and modulation recovery based on shape and topological sensitivity*. IFB-Report No. 13 (12/2007). Published in: Journal of Mathematical Imaging and Vision, Volume 35, Number 1 (September 2009). pp. 1-22.
4. M. Hintermüller and A. Laurain, *Optimal shape design subject to variational inequalities*. IFB-Report No. 24 (11/2008).
5. M. Hintermüller and M.H. Tber, *Local volatility estimation in American options: MPEC-view, C-stationarity and an active-set-Newton solver.* IFB-Report No. 19 (10/2008).
6. M. Hintermüller, M. Hinze and M.H. Tber, *An adaptive finite element Moreau-Yosida-based solver for a non-smooth Cahn-Hilliard problem*. IFB-Report No. 30 (08/2009).
7. M.H. Tber and A. Günther, *A Goal-Oriented Adaptive Moreau-Yosida Algorithm for Control- and State-Constrained Elliptic Control Problems*. IFB-Report No. 32 (12/2009).
8. Y. Dong, M. Hintermüller and M. Neri, *An Efficient Primal-Dual Method for L1-TV Image Restoration*. IFB-Report No. 27 (05/2009). To appear in: SIAM Journal on Imaging Sciences.

Relevant Publications:

9. R. H. Chan, Y. Dong and M. Hintermüller, *An Efficient Two-Phase L1-TV Method for Restoring Blurred Images with Impulse Noise*. IFB-Report No. 26 (03/2009).
10. Y. Dong, M. Hintermüller and M.M. Rincon-Camacho, *Automated regularization parameter selection in a multiscale total variation model*. IFB-Report No. 22 (11/2008).
11. M. Hintermüller and K. Kunisch, *PDE-constrained optimization subject to pointwise control and zero- or first-order state constraints*. IFB-Report No. 20 (10/2008). Published in: SIAM Journal on Optimization,Volume 20, Issue 3 (2009), pp. 1133-1156.
12. M. Hintermüller and I. Kopacka, *Mathematical Programs with Complementarity Constraints in function space: C- and strong stationarity and a path-following algorithm*. IFB-Report No. 11 (04/2008). Published in: SIAM Journal on Optimization, 20 (2009), pp. 868-902.
13. M. Hintermüller and I. Kopacka, *A smooth penalty approach and a nonlinear multigrid algorithm for elliptic MPECs*. IFB-Report No. 21 (11/2008).

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