Probability and Statistics

Syllabus for the TEMPUS–SEE PhD Course

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1 General goals of the course

The course should provide a high level overview on a wide range of statistical methods, data analysis, parameter estimation, test theory and on stochastic processes.
It is recommended that the course will be modified in the future in order to focus more on topics which are represented as research topics at the partner universities.

2 Prerequisites on the students side

Participating students should be familiar with basic notions of probability theory including measure theory and integration as well as basic statistics. Fundamentals of functional analysis are also required. Of course, English language proficiency is an absolute necessity.

3 Modules of the course

PART I: Probability theory and statistics (20 units)

Module	No. of units	Contents
I: Review of basic notions in probability theory	7	Fundamentals Bayes theorem, sensitivity, specifity Borel-Cantelli theorem Random variables, distribution functions, density Examples (Bernoulli, binomial etc.) Multi-dimensional normal distributions Expectation, variance, independence, correlation Moment generating functions, characteristic functions Convergence of random variables (almost surely, in probability, in the p-th mean, in distribution)
II: Laws of large numbers	4	Weak and strong law of large numbers Central limit theorem
III: Data analysis	3	Empirical distribution Quantil plots Regressions
IV: Hypothesis testing	6	Hypotheses Power of a test Maximum likelihood test t-test, F-test Parameter free tests

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PART II: Parameter estimation (20 units)

V: Point estimation theory	3	Output models Assumptions on the measurement process Properties of estimators Fisher information Cramer-Rao theorem
VI: Least squares estimators	3	Nonlinear least squares Nonlinear least squares Linear approximation Generalized least squares Numerical methods
VII: Maximum likelihood estimation	3	Normal data non-normal data
VIII: Bayesian estimation	3	Choice of prior distributions Posterior distributions Highest posterior density regions Normal approximation to posterior density
IX: Assymptotic theory	4	Introduction Least squares estimation Maximum likelihood estimation
X: Optimal experimental design	4	Introduction Generalized measurement procedures Probability measures on compact sets Optimal design criteria in terms of the Fisher information matrix

PART III: Stochastic processes (20 units)

XI: Markov chains	3	Construction and properties Examples Transience and recurrence Canonical decomposition Absorption probabilities Limit distributions
XII: Renewal theory	4	Counting renewals Renewal reward processes The Renewal Equation The Poisson Process Discrete renewal theory Stationary renewal processes Improper renewal equations

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XIII: Point processes	3	The Poisson Process Transforming Poisson Processes Max-stable and stable random variables More transformation theory Marking and thinning Variants of the Poisson Process The linear birth process as a point process
XIV: Continuous time Markov chains	2	Definitions and construction Stability and explosions The Markov property Stationary and limiting distributions Laplace transform methods
XV: Brownian motion	3	Introduction and construction of Brownian motion Properties of the standard Brownian motion The reflection principle The distribution of the maximum Brownian motion with drift
XVI: Martingales and semi-martingales	3	Introduction Stability properties Examples Stochastic integrals The quadratic variation of a semimartingale Change of variables (Ito's formula)
XVII: Stochastic differential equations	2	Existence and uniqueness of solutions Stability of stochastic differential equations Stochastic exponentials and linear equations
Total no. of units:	**60**	

4 Literature

[1] Asmussen, S., and Glynn, P. W., Stochastic Simulation, Algorithms and Analysis, Stochastic Modelling and Applied Probability Vol. 57, Springer-Verlag, New York 2007.

[2] Fedorov, V. V., Theory of Optimal Experiments, Academic Press, New York 1972.

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[4] Goodwin, G. C., and Payne, R. L., Dynamic System Identification: Experiment Design and Data Analysis, Mathematics in Science and Engineering Vol. 136, Academic Press, New York 1977.

[5] Lin'kov, Y. N., Lectures in Mathematical Statistics, Parts 1 and 2, Translations of Mathematical Monographs Vol. 229, American Mathematical Society, Providence, R.I., 2005.

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[7] Pázman, A., Foundations of Optimum Experimental Design, Mathematics and its Applications (East European Series), Reidel Publ. Comp., Dordrecht 1986.

[8] Protter, Ph. E., Stochastic Integration and Differential Equations, 2nd edition, Springer-Verlag, New York 2004.
[9] Resnick, S. F., Adventures in Stochastic processes, Birkhäuser, Basel 1992.
[10] Ross, S., Stochastic Processes, John Wiley, New York 1996.
[11] Schuss, Z., Theory and Applications of Stochastic Processes, an Analytical Approach, Applied Mathematical Sciences Vol. 170, Springer-Verlag 2010.
[12] Seber, G.A.F., and Wild, G. A., Nonlinear Regression, John Wiley & Sons, New York 1989.
[13] Shiryaev, A. N., Probability, 2nd ed., Graduate TExts in Mathematics Vol. 95, Springer-Verlag, New York 1996.

5 Teaching

The course should be accompanied by homework exercises which should require at most 2 of the afternoon sessions as indicated below. The major part of the afternoon session should be spent by working independently in teams on little projects on practical or pseudo-practical problems. The results also should be presented in the afternoon sessions. During the afternoon session the teacher should be available for questions respectively be present in order to get an impression on performance of the students. Homework exercises and projects for teamwork should also involve programming of algorithms respectively use of available software.

The course is planned for 4 weeks, each week from Monday till Friday. This implies that there will be 3 teaching units (45 minutes) per day. The following schedule is proposed for each day:

8:00 till 11:00:	three units with breaks in between;
11:30 till 12:30:	discussion with the teacher;
15:00 till 17:30:	work on homework exercises, work in teams on problems posed by the lecturer, presentation of results, respectively

Teachers for the course:

Part I	E. Pancheva
Part II	F. Kappel
Part III	W. Schappacher

6 Grading

The basis for grading is provided by the performance of students in the following items:

a) Exercises for *homework* involving also numerical computations will be regularly given in order to provide possibilities for a better understanding of the material presented in the course.
b) *Team projects* and presentation of results.
c) An *oral examination* concerning the course.

The oral examination could consist of several parts taken at different times and should give the lecturer an impression on how well the student has understood the material of the course.

In order to obtain the grade for the course the following weights will be used for the items a), b) and c) from above:

Homework exercises	20%
Team projects	50%
Oral examination	30%