Dynamical Systems
Syllabus for the TEMPUS - SEE PhD Course

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1. General goals of the course

The goal of the course is to give to the students a basic knowledge about dynamical systems and a qualitative insight to differential equations.

2. Prerequisites

Metric and topological spaces; linear algebra; ordinary differential equations; Familiarity with "Mathematica"

3. Course modules

Units	Contents
20	**Module I. Topological dynamical systems (Senada Kalabusic)**
1	Discrete dynamical systems. Difference equations. Population growth model. Linear dynamical systems.
2	Maps. Arnold's cat map, Baker's map, Circle map, Henon map, Horseshoe map, Logistic map, Duffing map, Complex quadratic map,
2	Fixed (Equilibrium) points. Periodic points. Graphical iteration and stability. Fixed points for qudaratic family
2	Limit sets. $\alpha-$limit set. $\omega-$limit set. Nonwandering point. Invariant set
2	Invariant Cantor sets for the quadratic family.
3	Conjugacy and structural stability.
2	Homeomorphisms of the circle. Rotation number. Examples.
2	The period doubling. $2-$cycles. 2^2-cycles. Beyond μ_∞ .
4	Li-Yorke theorem. Sharkovski ordering. Sharkovski theorem. Examples for Sharkovski theorem.
20	**Module II. Continuous Dynamical Systems (Ognyan Christov)**
3	Review of basic concepts and theorems in ODE. Vector fields, flows, linear systems, fixed points, linearization, phase portraits, stability
1	Floquet theorem, logarithm of the matrix
2	Poincare maps. Examples. Duffng equation
2	Equivalence, equivalence of linear systems. Hartman - Grobman's theorem.
3	Limit sets. Poincare – Bendixson theorem.
2	Normal forms. Resonances. Poincare theorem.
3	TCenter manifolds. Approximate computations.
3	Bifurcation of fixed points. A zero eigenvalue. Hopf bifurcation.
1	Attractors. Lorenz, Rosler and Chua attractors.
20	**Module III. Chaos (Donco Dimovski)**
3	Symbolic dynamics, Smale Horseshoe map. invariant set
3	Shift map. The structure of the space of symbol sequences.
3	Conley Moser conditions for chaos.
2	Liapunov exponents
2	Sensitivity to initial conditions, topological transitivity.
2	Density of periodic orbits
2	Chaos and strange atractors
2	Linking of periodic orbits. Templates.
1	Synchronization. Coupling of tvo dynamical systems.

4 Lecturing

Explaining the basic notions and definitions. Stating the facts and give ideas about their proofs, and assign the proofs to the students for homework. Students will present their proofs. Students will do computer simulations of the dynamics for specific dynamical systems, in order to obtain some conclusions of its behavior, depending on some parameters.

5 Grading

a) Homework - proving theorems given by the lecturers, and presenting their proofs.
b) Homework - computer simulations of the dynamics for specifc dynamical systems, in order to obtain some conclusions of its behaviour, depending on some parameters.
c) A final (not formal) oral exam at the end of each module.
The weights of a), b), c) for the final grade:
a) 50 % b) 30 % c) 20 %

6 References

[1] V. I. Arnold, "Ordinary differential equations", various editions.
[2] S. Wiggins, Introduction to applied nonlinear dynamical systems and chaos, Springer, 2003.
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[4] S. Lynch, Dynamical systems with applications using Mathematica, BirkhÄauser, 2007.
[5] G. Teschl, Ordinary Differential Equations and Dynamical Systems, Springer, 2009.
[6] M. Hirsh, S. Smale, R. Devaney, Differential equations, dynamical systems and an introduction to chaos, Elsevier, 2004.
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