FEM⚭BEM - The Joint Power of the Domain Decomposition FEM/BEM and the Xplorer[1]

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Keywords : Parallel Algorithms; Iterative Solvers; Domain Decomposition; Nonlinear Magnet Field Computations; Exterior Problems; Elasticity

At the present time high effort is being paid in both developing and implementing parallel algorithms. The program FEM⚭BEM presented here is part of this ongoing work and unites several results concerning the development of preconditioning techniques, domain decomposition methods and the coupling of Finite Element Methods and Boundary Element Methods [3, 8]. The latter is of special interest if exterior problems are to be solved.
Although the present version of FEM⚭BEM will be developed further it can already be used to solve real-life problems arising in magnetostatics, or in solid mechanics. The main goal of FEM⚭BEM is to provide an efficient solver for linear and nonlinear systems arising from the discretization of the partial differential equations describing the problem under consideration. These solvers are based on a non-overlapping domain decomposition and multilevel methods [1, 2].
In the following we present some examples performed on a 16-processor Xplorer system of Parsytec or on a 128-processor PowerGC system.

1 Electromagnet

Here, we want to simulate the magnetic field caused by an electromagnet as shown in Figure 1. The problem can be described formally by the following partial differential equation

$$-\mathrm{div}(\nu(x,|\nabla u|)\nabla u(x)) \;=\; f(x), \quad x \in R^2 \qquad \text{and} \qquad |u(x)| \;\to\; 0 \quad \text{for } |x| \to \infty,$$

where the unknown u represents the z-component of the vector potential $\vec{A}$ introduced in the Maxwell-Equations by $\vec{B} = \mathrm{rot}\vec{A}$ [5]. The domain is unbounded. Thus, we have to impose

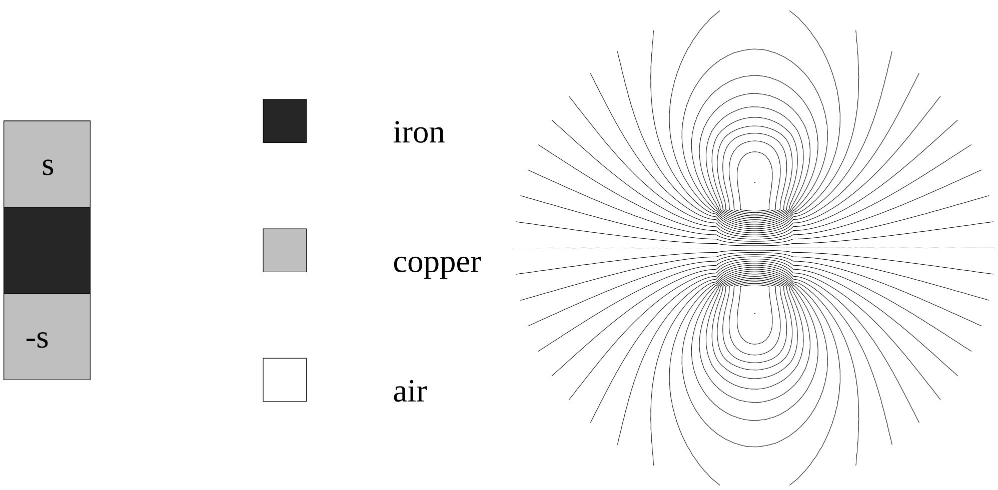

Figure 1: Three FEM-subdomain and one BEM exterior domain. Equipotential lines.

Sommerfeld's conditions on the behaviour of $u(x)$ for $|x| \to \infty$. That is why we are going

[1] This work has been supported by the German Research Foundation DFG within the Priority Research Programme "Boundary Element Methods" under grant La 767/1-3

to use a coupled Finite Element and Boundary Element model based upon domain decomposition into inner subdomains and one exterior boundary element subdomain [6, 8]. In the inner subdomains, we can use either FEM or BEM (the latter only if certain conditions are fulfilled).
Using 4 processors of the Xplorer, the program needs only 30 seconds to solve the above described 16.129 nonlinear finite-element–boundary element equations with a relative accuracy of 10^{-6}.

2 The Electric Motor

As a more challenging example for an exterior problem we consider an electric motor. It consists of four permanent magnets of alternating magnetization, a rotor with 4 wings, regions of air and an iron coating. Figure 2 shows the mesh being used on the left and the equipotential lines of A_z have been drawn on the right.
We have to solve the same partial differential equation as in the foregoing Example 1 with a different right-hand side (wings and permanent magnets) and homogeneous Dirichlet boundary conditions [4, 5]. Using 64 subdomains with 1.511.960 unknowns the nonlinear problem was solved in 97 seconds on the PowerGC.

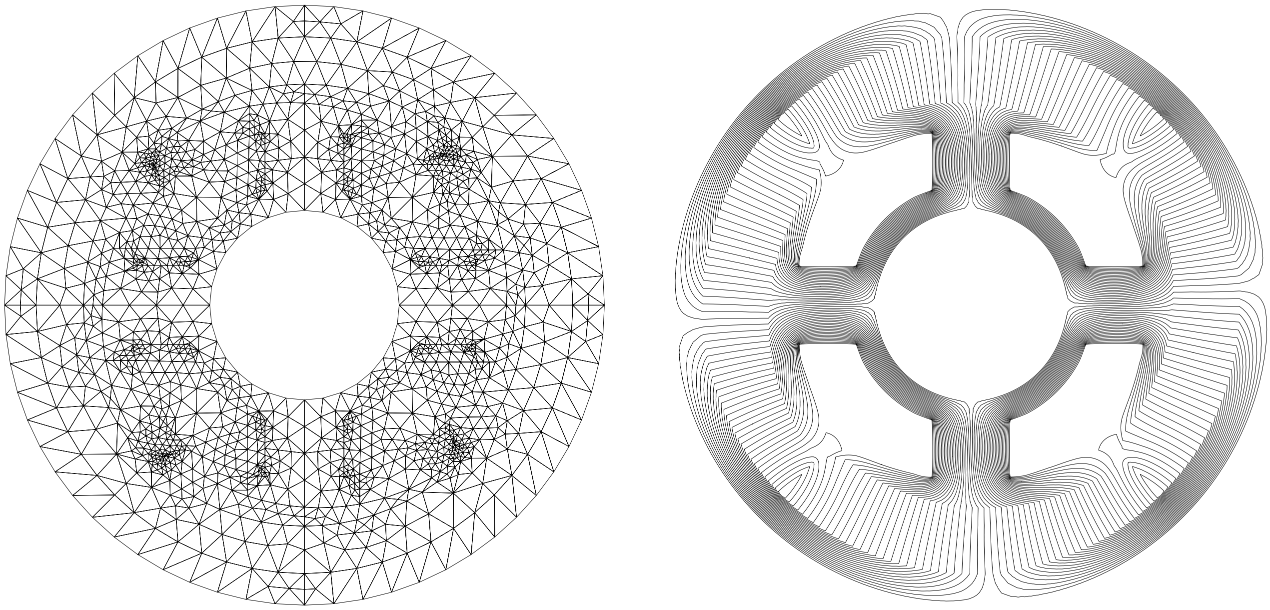

Figure 2: The FEM discretization of the motor and the equipotential lines of the solution.

3 Elasticity

The code can also be applied to problems of elasticity described in terms of the displacement $\underline{u}$, the inner forces $\underline{f}$, the traction $\underline{t}$ and the stress tensor $\sigma = \sigma(\underline{u})$ (Hook's law)

$$\begin{aligned} -\mu \triangle \underline{u}(\underline{x}) \; - \; (\lambda + \mu) \operatorname{grad}(\operatorname{div} \underline{u}(\underline{x})) &= \underline{f}(\underline{x}) && \text{in } \Omega \\ u_\imath(\underline{x}) &= g_\imath(\underline{x}) && \text{on } \Gamma_{1,\imath} \quad \imath=1,2 \\ \sum_{\jmath=1}^{3} \sigma_{\imath,\jmath}\eta_\jmath &= t_\imath(\underline{x}) && \text{on } \Gamma_{2,\imath} \quad \imath=1,2 \end{aligned} .$$

In a first example, we consider the deformation of a thin linearly elastic plate with a hole in the middle . We suppose the plate to be a homogeneous isotropic elastic body. The tension forces act on the left and right border of the plate into opposite directions ([7] pp. 230ff). Taking into account the symmetries, we can reduce the calculation domain to a quarter of this plate. Using a triangular linear finite element discretization, we solved the resulting really

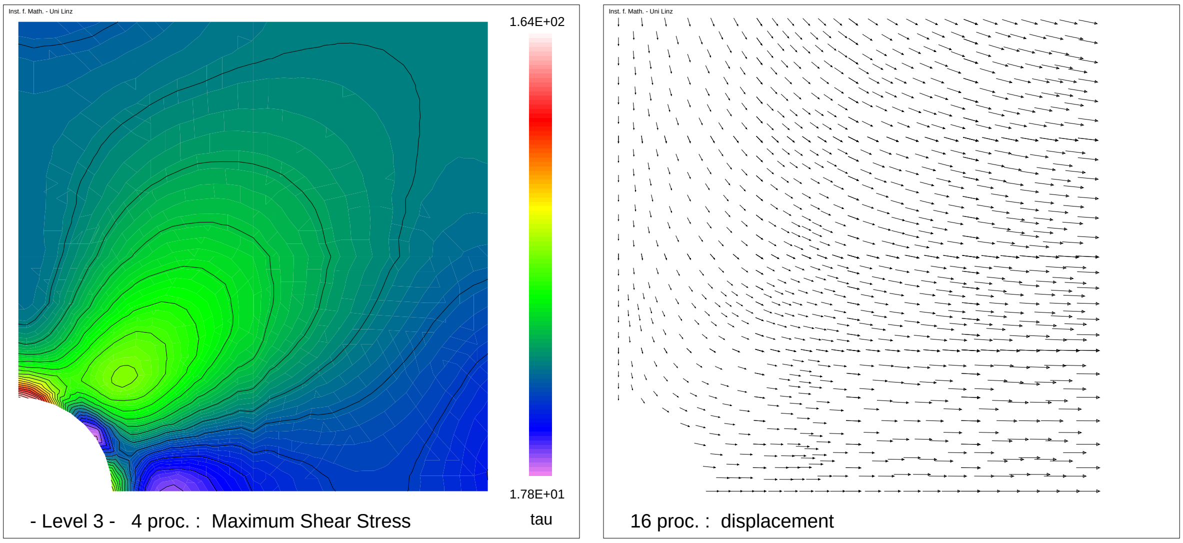

Figure 3: Maximum shear stress and displacement vectors of the thin plate

large-scale system of 447.000 finite element equations within one minute on our 16 processor PowerXplorer. The adequate domain decomposition boundary element model yielding 16.083 degrees of freedom could be solved within the same CPU-time on the same configuration of processors.

A more advanced problem consists in computing the deformation of an embankment-type dam (concrete) for which we use FEM inside the dam and BEM in the rock on the ground. This combination of the two methods avoids large FEM domains in the rock area or boundary conditions influencing the behavior of the stress on the interface between dam and rock [9].

4 Some Remarks

During the last 4 months we have changed from a MultiCluster-II system to an Xplorer system. These are our observations :

- We obtained a much better improvement (measured by time) of our algorithms then expected by the MFLOP rates. Using exactly the same source code as on the MultiCluster-II we achieved a factor 30 for problems which use approximately 8 Mbyte memory per processor. From our point of view, we gained this additional improvement by the use of our own vector library calls which may be hold in the cache during the operations.
- The optimizer presently used is very good and fast. Problems occur when EXTERNAL declarations in FORTRAN are used.
- The time for loading a program into the parallel machine was rapidly decreased.
- The measured communication performance is just twice as good. In our numerical algorithms this bottleneck can just be observed when the usage of memory is less then one per cent per node.

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