The number of invariant subspaces under a linear operator on finite vector spaces

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Let V be an n-dimensional vector space over the finite field $\mathbb{F}_q$ and T a linear operator on V. For each $k \in \{1,\ldots,n\}$ we determine the number of k-dimensional T-invariant subspaces of V. Finally, this method is applied for the enumeration of all monomially nonisometric linear (n,k)-codes over $\mathbb{F}_q$.

The problem

$\mathbb{F}_q$: finite field of cardinality q

V: vector space of dimension n over $\mathbb{F}_q$, $V = \mathbb{F}_q^n$

T: a linear operator on V

U: a subspace of V is T-invariant if $TU \subset U$

Determine the polynomial

$$\sigma(T) = \sum_{k=0}^{n} \sigma_k(T) x^k \in \mathbb{Q}[x],$$

where $\sigma_k(T)$ is the number of k-dimensional, T-invariant subspaces of V.

Examples

Let $V = \mathbb{F}_3^3$.

1. $T = \mathrm{id}_V$ or $T = 0$. Each subspace is T-invariant. $\sigma(T) = 1 + 13x + 13x^2 + x^3$, where $13 = \left[\binom{3}{1}\right](3) = \left[\binom{3}{2}\right](3)$ is a Gauss polynomial or q-binomial coefficient.

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2. $T(x_1, x_2, x_3) = (x_2, x_3, x_1)$ a cyclic shift. $\sigma(T) = 1 + x + x^2 + x^3$. The T-invariant subspaces form a chain $\{0\} \subset \langle (1,1,1) \rangle \subset \langle (1,1,1), (0,1,2) \rangle \subset V$.

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3. $T(x_1, x_2, x_3) = (x_1, x_2, x_3) \cdot \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = (2x_3, x_1 + x_3, x_2)$. $\sigma(T) = 1 + x^3$. The minimal polynomial of T is $x^3 + 2x + 1$. It is irreducible over $\mathbb{F}_3$.

4. $T(x_1,x_2,x_3) = (x_1,x_2,x_3) \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = (x_1+x_2,x_2,2x_3).$

The matrix is in normal form containing a hypercompanion matrix of $x-1$ and a companion matrix of $x-2$.

There are two invariant subspaces

$$V_1 := \{(x_1,x_2,0) \mid x_1, x_2 \in \mathbb{F}_3\} \text{ and } V_2 := \{(0,0,x_3) \mid x_3 \in \mathbb{F}_3\}.$$

Let T_i be the restriction of T to V_i, then

$$\sigma(T) = \sigma(T_1)\sigma(T_2) = (1+x+x^2)(1+x) = 1+2x+2x^2+x^3.$$

The lattice of invariant subspaces

The T-invariant subspaces of V form a lattice, the ***lattice*** $L(T)$ ***of*** T***-invariant subspaces***.

Brickman and Fillmore (1967): the lattice $L(T)$ is self-dual, which means that the coefficients of $\sigma(T)$ satisfy $\sigma_k(T) = \sigma_{n-k}(T)$ for $0 \leq k \leq n$.

A vector space as a module

T a linear operator on V

V is a left $\mathbb{F}_q[x]$-module by

$$\mathbb{F}_q[x] \times V \to V$$

$$(f, v) \mapsto fv := \sum_{i=0}^{r} a_i T^i v,$$

where $f = \sum_{i=0}^{r} a_i x^i \in \mathbb{F}_q[x]$.

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The monic polynomial of least degree which annihilates v is called the ***minimal polynomial*** of v.

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There exists a monic polynomial $g \in \mathbb{F}_q[x]$ of least degree which annihilates all vectors in V. It is called the ***minimal polynomial*** of T.

Primary decomposition

$g = \prod_{i=1}^{s} f_i^{c_i}$ factorization of the minimal polynomial into irreducible divisors

$V_i := \left\{ v \in V \mid f_i^{c_i} v = 0 \right\}$ is a T-invariant subspace

$$V = \bigoplus_{i=1}^{s} V_i$$

Primary decomposition

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$$V = \bigoplus_{i=1}^{s} V_i$$

let T_i be the restriction of T to V_i

Brickman and Fillmore (1967): the lattice $L(T)$ is the direct product of the lattices $L(T_i)$, i. e.

for each $U \in L(T)$ there exists exactly one $(U_1, \dots, U_s) \in \prod_{i=1}^{s} L(T_i)$, so that $U = U_1 \oplus \cdots \oplus U_s$.

Therefore, $\sigma(T) = \prod_{i=1}^{s} \sigma(T_i)$.

Consequently it is enough to study the lattices $L(T_i)$ of the primary components V_i, $1 \leq i \leq s$.

Cyclic vector spaces

For $v \in V$ let $[v] := \mathbb{F}_q[x]v = \{fv \mid f \in \mathbb{F}_q[x]\}$ be the ***cyclic subspace generated by*** v.

It is T-invariant. It is the smallest T-invariant subspace of V containing v. Its dimension is the degree of the minimal polynomial of v.

U is called ***cyclic*** if there exists some $v \in U$, so that $U = [v]$.

Decomposition of a primary space into cyclic subspaces

V an n-dimensional vector space, with minimal polynomial f^c, f irreducible

$$V = \bigoplus_{i=1}^{r} U_i$$

$U_i = [v_i] \simeq \mathbb{F}_q[x]/I(f^{t_i})$ and $c = t_1 \geq \ldots \geq t_r \geq 1$.

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Define the ***height*** of both v and $[v]$ by

$$h([v]) := h(v) := \frac{\dim[v]}{\deg f}.$$

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Brickman and Fillmore (1967): Lattice of each summand is a chain $U_i = [v_i] \supset [fv_i] \supset \ldots \supset [f^{t_i-1}v_i] \supset \{0\}$.

The elements of $[v_i] \setminus [fv_i]$ generate U_i.
These are $Q^{t_i} - Q^{t_i-1}$ vectors, where $Q = q^{\deg f}$.

Species of a primary vector space

In general, the decomposition of a primary vector space V as a direct sum of cyclic subspaces is **not unique**.

Consider $V = \bigoplus_{i=1}^{r} [v_i]$ from above.
The ***species of this decomposition*** is the vector $\lambda = (\lambda_1, \ldots, \lambda_c)$ where λ_j is the number of summands $[v_i]$ of height j, i. e.

$$\lambda_j = |\{i \in \{1, \ldots, r\} \mid h(v_i) = j\}|.$$

Consequently, $\sum_{j=1}^{c} \lambda_j = r$ and $\sum_{j=1}^{c} j\lambda_j \deg f = \dim V = n$.

The species of two different decompositions of V as a direct sum of cyclic subspaces are the same.
We call $(\lambda_1, \ldots, \lambda_c)$ the ***species of*** V.

Number of all subspaces

$V = \mathbb{F}_q^n$, $T = \mathrm{id}_V$, the minimal polynomial of T is $f = x - 1$, thus $c = 1$. Each 1-dimensional subspace is a cyclic one, hence $\langle v \rangle = [v]$ for all $v \in V$. The species of V is $\lambda = (n)$. Let $e^{(i)}$ be the i-th unit vector in $\mathbb{F}_q^n$, $1 \le i \le n$, then two decompositions of V as a direct sum of cyclic subspaces are e. g.

$$V = \bigoplus_{i=1}^{n} [e^{(i)}] = \bigoplus_{i=1}^{n} [e^{(1)} + \cdots + e^{(i)}].$$

Each k-dimensional subspace of V is T-invariant and has the species $\mu = (k)$. Thus the number of k-dimensional T-invariant subspaces of V is

$$\left[\binom{n}{k}\right](q) = \prod_{i=0}^{k-1} \frac{q^n - q^i}{q^k - q^i}. \qquad (*)$$

The nominator in $(*)$ determines the number of all k-tuples $(u_1,\ldots,u_k)$ in V^k so that the u_i, $1 \le i \le k$, are linearly independent. Hence, it is the number of all k-tuples $(u_1,\ldots,u_k)$ in V^k so that $h(u_i) = 1$, $1 \le i \le k$, and that the sum of the cyclic spaces $[u_i]$, $1 \le i \le k$, is direct. Therefore, the sum $\bigoplus_{i=1}^{k} [u_i]$ has the species μ.

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Consider an arbitrary k-dimensional subspace U of V. Then its species is μ. The denominator in $(*)$ is the number of all k-tuples $(u_1,\dots,u_k)$ in U^k so that the u_i, $1 \le i \le k$, are linearly independent. Hence, it is the number of all k-tuples $(u_1,\dots,u_k)$ in U^k so that $h(u_i) = 1$, $1 \le i \le k$, and that the sum of the cyclic spaces $[u_i]$, $1 \le i \le k$, is direct. Therefore, the sum $\bigoplus_{i=1}^k [u_i]$ is equal to U and has the species μ.

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This method was generalized in *Séguin (1996)* to the computation of the number of T-invariant subspaces of a primary space where the minimal polynomial of T is just irreducible, i. e. in our terminology $c = 1$.

Subspaces of given species

Let V be a primary vector space of species $\lambda = (\lambda_1, \ldots, \lambda_c)$, $\lambda_c \neq 0$, and let μ be the species of a subspace of V. How to construct all subspaces of species μ?

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Let V be a primary vector space of species $\lambda = (\lambda_1, \ldots, \lambda_c)$, $\lambda_c \neq 0$, and let μ be the species of a subspace of V. How to construct all subspaces of species μ?

Lemma. Consider some $t \in \{1, \ldots, c\}$, and some $v \in V$ with $h(v) = t$. Let U be a T-invariant subspace of V of species $\nu = (\nu_1, \ldots, \nu_c)$ so that $\nu_i = 0$ for $i < t$. Then $U \cap [v] = \{0\}$ if and only if $h(v - u) \geq t$ for all $u \in U$ (or equivalently, $v \neq u + w$ for all $u \in U$ and all $w \in V$ with $h(w) < t$).

Algorithm

Consider V a space of species λ and μ the species of a subspace of V. Let $s = \sum_{i=1}^{c} \mu_i$. Now we describe an algorithm for determining all sequences $(u_1, \ldots, u_s) \in V^s$, so that $h(u_1) \geq \ldots \geq h(u_s)$, the sum $[u_1] + \cdots + [u_s]$ is direct and the species of $[u_1] \oplus \cdots \oplus [u_s]$ is μ.

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1) Let $k_1 := \max\{j \in \{1,\ldots,c\} \mid \mu_j \neq 0\}$.

2) Determine $u_1 \in V$ so that $h(u_1) = k_1$.

3) Let $U_1 := [u_1]$ and let $\nu^{(1)}$ be the species of U_1. Let $i := 1$.

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4) If $\nu^{(i)} \neq \mu$ let $k_{i+1} := \max\{j \in \{1, \dots, c\} \mid \mu_j \neq \nu_j^{(i)}\}$, else goto 7).

5) Determine $u_{i+1} \in V$ so that $h(u_{i+1}) = k_{i+1}$ and $U_i \cap [u_{i+1}] = \{0\}$.

6) Let $U_{i+1} := U_i \oplus [u_{i+1}]$ and let $\nu^{(i+1)}$ be the species of U_{i+1}.
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7) Output $(u_1, \dots, u_s)$ where $s = \sum_{i=1}^{c} \mu_i$.

How many possible choices for u_1 in 2)?

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Lemma. For $t \in \{1, \ldots, c\}$ the number of vectors of height t is equal to

$$\alpha_t(\lambda) = \frac{Q^t - Q^{t-1}}{Q-1} Q^{(t-1)(l_t-1)} (Q^{l_t} - 1) \prod_{i=1}^{t-1} Q^{i\lambda_i}$$

where $l_t := \lambda_t + \cdots + \lambda_c$ and $Q = q^{\deg f}$.

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Lemma. For $t \in \{1,\ldots,c\}$ the number of vectors of height t is equal to

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Lemma. Consider some $t \in \{1,\dots,c\}$. Let U be a T-invariant subspace of V of species $\nu = (\nu_1,\dots,\nu_c)$ so that $\nu_i = 0$ for $i < t$. Let $Q := q^{\deg f}$. Then there exist

$$\beta_t(\lambda,\nu) = \alpha_t(\lambda) - \alpha_t(\nu) \prod_{i=1}^{t-1} Q^{i\lambda_i} Q^{(t-1)\sum_{i=t}^{c}(\lambda_i - \nu_i)}$$

vectors $v \in V$ so that $h(v) = t$ and $U \cap [v] = \{0\}$.

Main results

Theorem. Let V be a primary vector space of species $\lambda = (\lambda_1, \dots, \lambda_c)$. The number of different subspaces of V of species μ is equal to

$$\frac{\gamma(\lambda,\mu)}{\gamma(\mu,\mu)},$$

where

$$\gamma(\lambda,\mu) := \alpha_{k_1}(\lambda) \prod_{i=1}^{s-1} \beta_{k_{i+1}}(\lambda, \nu^{(i)}).$$

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Which species occur as species of subspaces of V?

Theorem. Let V be a primary vector space of species $\lambda = (\lambda_1, \dots, \lambda_c)$. The sequence $\mu = (\mu_1, \dots, \mu_c)$ is the species of a subspace of V if and only if $\sum_{i=j}^{c} \mu_i \le \sum_{i=j}^{c} \lambda_i$ for all $j \in \{1, \dots, c\}$.

Monomial isometry classes of linear codes

For $1 \leq k \leq n$ a linear (n,k)-code C over $\mathbb{F}_q$ is a k-dimensional subspace of $\mathbb{F}_q^n$. Two linear (n,k)-codes are called monomially isometric if there exists a monomial matrix M, i. e. a regular $n \times n$-matrix which has in each row and in each column exactly one nonzero component, so that $C_2 = C_1 M^{-1} = \left\{ c \cdot M^{-1} \mid c \in C_1 \right\}$.

The monomial matrices form the group $M_n(q)$, the full monomial group over the multiplicative group $\mathbb{F}_q^*$, which is isomorphic to the wreath product $\mathbb{F}_q^* \wr S_n$, where S_n is the symmetric group on $\{1,\ldots,n\}$.

Then the multiplication of a code C with M^{-1} from the right describes an action of the group $M_n(q)$ on the set $\mathcal{U}_{nk}(q)$ of all (n,k)-codes over $\mathbb{F}_q$.

The isometry class of the code C is then the orbit $\{CM \mid M \in M_n(q)\}$ of C. Therefore, using the Lemma by Cauchy-Frobenius, the number of monomially nonisometric linear (n,k)-codes over $\mathbb{F}_q$ is the average number of fixed points in $\mathcal{U}_{nk}(q)$ for all monomial matrices.

Each monomial matrix M yields a linear operator T_M on $\mathbb{F}_q^n$ defined by $v \mapsto v \cdot M$. A linear code C is a fixed point of $M \in M_n(q)$ if and only if C is T_M-invariant. Thus the number of monomially nonisometric linear (n,k)-codes over $\mathbb{F}_q$ is the average number of T_M-invariant k-dimensional subspaces of $\mathbb{F}_q^n$ for all $M \in M_n(q)$.

This method is implemented in GAP and SYMMETRICA. Our results allowed to confirm previously computed data and to enlarge the sets of parameters (n,k,q) where we are able to determine the numbers of nonisometric codes explicitly. From the description above it is clear that this method is the natural way for enumerating monomially nonisometric codes.

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