

DISCRETE AND ALGEBRAIC STRUCTURES

Master Study Mathematics

MAT.402

Winter Semester 2015/16

(Due to organisational reasons register for the course at TU and KFU)

Edited by Mihyun Kang

TU Graz

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Part I

Modules

Chapter 1

Lectures (MAT.402)

1.1 Enumerative Combinatorics

Mihyun Kang

5 October 2015 – 13 October 2015 (L1 – L5)

Mondays and Tuesdays, **08:00 – 09:45**, BE01

⇒ 2nd Semester: Analytic combinatorics

1.2 Graph Theory

Mihyun Kang

19 October 2015 – 10 November 2015 (L6 – L10)

Mondays and Tuesdays, **08:00 – 09:45**, BE01

⇒ 2nd Semester: Advanced and algorithmic graph theory

⇒ 3rd Semester: Probabilistic method in combinatorics and algorithmics

1.3 Multilinear Algebra

Karin Baur

23 November 2015 – 19 January 2016 (L11 – L22)

Mondays and Tuesdays, 08:15 – 09:45, BE01

1.4 Rings and Moduls

Karin Baur

23 November 2015 – 19 January 2016 (L11 – L22)

Mondays and Tuesdays, 08:15 – 09:45, BE01

Chapter 2

Exercises

(MAT.403-1 and MAT.403-2)

2.1 Enumerative Combinatorics

Philipp Sprüssel

12 October 2015 – 20 October 2015

2.2 Graph Theory

Philipp Sprüssel

26 October 2015 – 20 November 2015

2.3 Multilinear Algebra

Karin Baur

30 November 2015 – 29 January 2016

2.4 Rings and Moduls

Karin Baur

30 November 2015 – 29 January 2016

Chapter 3

Written Examinations

3.1 1st Examination

1 – 5 February 2016

3.2 2nd Examination

April 2016 ??

3.3 3rd Examination

June 2016 ??

Chapter 4

Prerequisites

4.1 Discrete Mathematics

(1st Semester of Bachelor Study)

4.2 Linear Algebra 1

(1st Semester of Bachelor Study)

4.3 Linear Algebra 2

(2nd Semester of Bachelor Study)

4.4 Introduction to Algebra

(4th Semester of Bachelor Study)

4.5 Probability Theory

(4th Semester of Bachelor Study)

4.6 Introduction to Complex Analysis

(5th Semester of Bachelor Study)

Part II

**Standard Methods of
Enumerative Combinatorics**

Chapter 1

Basics

Chapter 2

Recursions

2.1 Recursions

2.2 Linear recurrences

2.3 Non-linear recurrences