DISCRETE AND ALGEBRAIC STRUCTURES, WS 2016/17

Exercise Sheet 9

Problems due 16.12.2016

(9.1) (1) Let $E_i = \{e_{i1}, \dots, e_{in_i}\}$ be a basis of V_i, $i = 1, \dots, m$. Define

$$\varphi : (V_1 \otimes \cdots \otimes V_k) \times (V_{k+1} \otimes \cdots \otimes V_m) \to V_1 \otimes \cdots \otimes V_m$$

by $\varphi(e_{1i_1} \otimes \cdots \otimes e_{ki_k}, e_{k+1i_{k+1}} \otimes \cdots \otimes e_{mi_m}) = e_{1i_1} \otimes \cdots \otimes e_{mi_m}$ (with bilinear extension). Show that φ is the tensor map satisfying

$$\varphi(v_1 \otimes \cdots \otimes v_k, v_{k+1} \otimes \cdots \otimes v_m) = v_1 \otimes \cdots \otimes v_m$$

(2) (Show that $\mathbb{Z}/2\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}/3\mathbb{Z} = 0$ – tensor products of modules over rings!)

(9.2) Let U, V, W be vector spaces over $\mathbb{C}$. Show that there is an isomorphism $(U \oplus V) \otimes W \cong (U \otimes W) \oplus (V \otimes W)$ sending $(u \oplus v) \otimes w$ to $(u \otimes w) \oplus (v \otimes w)$

(9.3) (1) Let V be a complex vector space. Show that there is an isomorphism

$$V \otimes \mathbb{C} \cong V$$

(2) Let V be a complex vector space. Let W be a complex vector space of dimension n. Show that

$$V \otimes W \cong \underbrace{V \oplus \cdots \oplus V}_{n \text{ copies}}$$

(9.4) (1) Let $K[x]$ be the polynomials in x over the field K, $K[x, y]$ be the polynomials over K in x and y. Show that

$$K[x] \otimes K[y] \cong K[x, y] \quad \text{via } f(x) \otimes g(y) \mapsto f(x)g(y)$$

(2) Show that $V \otimes W \cong W \otimes V$ naturally (i.e. independent of the choice of basis).