Preconditioning Techniques for the Bidomain Equations

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Summary. In this work we discuss parallel preconditioning techniques for the bidomain equations, a non-linear system of partial differential equations which is widely used for describing electrical activity in cardiac tissue. We focus on the solution of the linear system associated with the elliptic part of the bidomain model, since it dominates computation, with the preconditioned conjugate gradient method. We compare different parallel preconditioning techniques, such as block incomplete LU, additive Schwarz and multigrid. The implementation is based on the PETSc library and we report results for a 16 node HP cluster. The results suggest the multigrid preconditioner is the best option for the bidomain equations.

1 Introduction

The set of bidomain equations[6] is currently the mathematical model which best reflects the electrical activity of the heart. The non-linear partial differential equations (PDEs) model both the intracellular and extracellular domains of cardiac tissue from an electrostatic point of view. The coupling of the domains is done via non-linear models describing the current flow through the cell membrane. Such models are based on experimental data that quantify different ionic contributions, as first proposed by Hodgkin and Huxley[4].

Unfortunately, the bidomain equations are computationally very expensive. Modern membrane models involve more than 20 non-linear equations. One way to avoid the solution of a large non-linear PDE system at every time step is to use an operator splitting approach[22, 21, 7, 23]. The numerical solution reduces the solution of a parabolic equation, a non-linear system of ordinary differential equations, and an elliptic system. It is the latter which dominates computation[22].

An efficient way of solving the large linear algebraic system that arises from the discretization of the bidomain equations has been a topic of research since 1994 [5]. Many different approaches have been employed with