A model of Neuromuscular Reflex Motion

This chapter describes the dynamics of neuromuscular reflex motion, which may give insight into the status of patients who have neurological disorders. The patient is seated comfortably and the upper arm remains in a fixed horizontal position, whereas the forearm is allowed to move only in the vertical plane. The initial angle between the forearm and upper arm is 135°. Then, at time t=0, a distortion, an additional weight is abruptly added to the original load. Changes in angular motion, θ(t), of the forearm are recorded during and after the quick release of the weight. The mathematical model used to interpret the results of this test is based on the work of Soechting et al. (1971)

Limb Dynamics

A schematic diagram of the forearm is given in Figure 1. M_x represents the change in external moment acting on the limb about the elbow joint. In this experiment it is a step function. M(t) represents the net muscular torque exerted in response to the external disturbance. of motion:

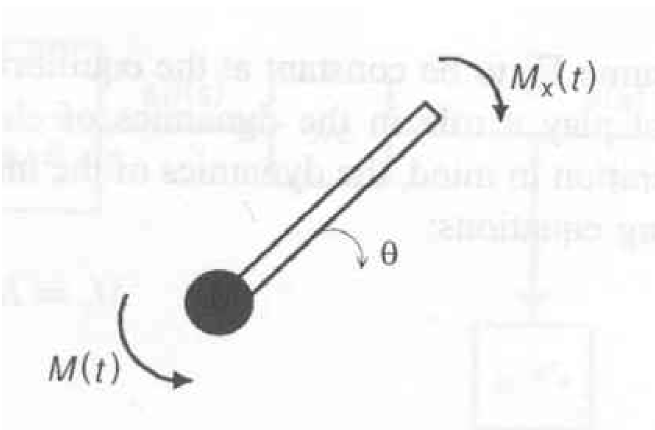

Figure 1: Components of the neuromuscular reflex model: limb dynamics

Neglecting the weight of the forearm itself, applying Newton's second law yields the following equation:

$$M_x(t) - M(t) = J\ddot{\Theta} \qquad \text{Eq.1}$$

where J is the inertia of the forearm about the elbow joint.

Muscle Model

For simplicity a net muscular torque is assumed, this is illustrated in Figure 2. Accordingly, the "displacements" that result are in fact angular changes, θ and θ_1.

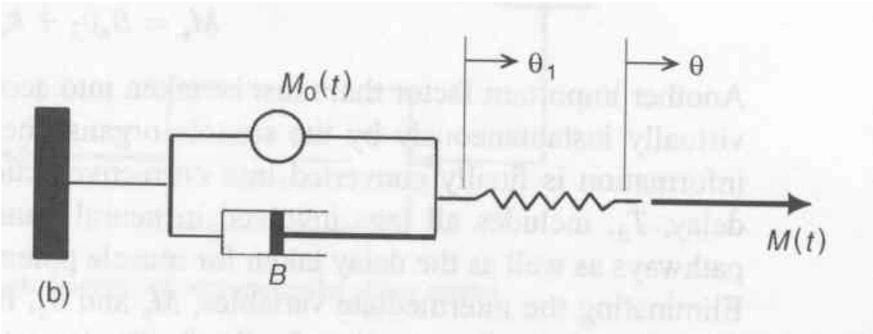

Figure 2: Components of the neuromuscular reflex model: muscle model

The parameter k is the muscle stiffness, and the parameter B is the viscous damping parameter. The equations of motion for the muscle model are:

$$M(t) = k(\Theta - \Theta_1) \quad \text{Eq.2}$$

and

$$M(t) = M_0(t) + B\dot{\Theta}_1 \quad \text{Eq.3}$$

where $M_0(t)$ is the torque exerted by the muscle under isometric conditions. $M_0(t)$ is represented as a function of time, since it is dependent on the pattern of firing of the alpha motorneurons.

Plant Equations

Combing Eq.1-Eq.3 we obtain an equation of motion that characterizes the dynamics of the plant: describing how θ would change due to the torque exerted by the external disturbance M_x, which leads to the following dynamic equation:

$$\frac{BJ}{k}\dddot{\Theta} + J\ddot{\Theta} + B\dot{\Theta} = M_x(t) - M_0(t) \quad \text{Eq.4}$$

Muscle Spindle model

This model describes the dynamics by which changes in θ are transduced at the level of the muscle spindles into afferent neural signals. The latter travel to the spinal cord, which sends out efferent signals to the contractile machinery of the muscle to generate $M_0(t)$.

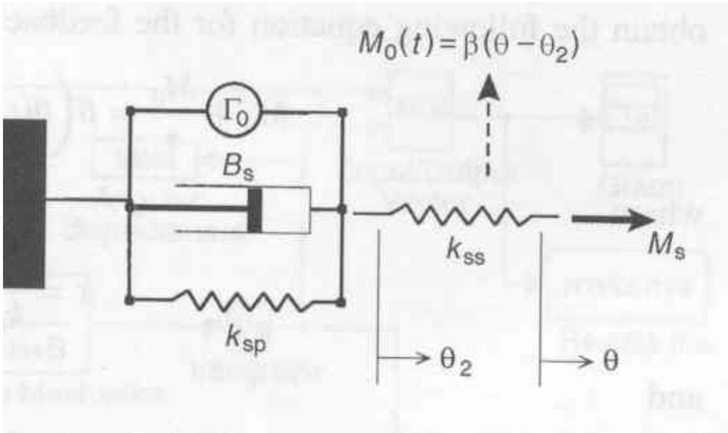

Figure 3: Components of the neuromuscular reflex model: muscle spindle

Assuming that the neural output of the spindle is proportional to the amount by which its nuclear bag region is stretched this leads to:

$$M_0(t) = \beta(\Theta - \Theta_2) \quad \text{Eq.5}$$

Figure 3 is showing the whole spindle model. K_{sp} and B_s are parameters that represents the elastic stiffness and viscous damping properties of the pole region of the spindle, while k_{ss} represents the elastic stiffness of the nuclear bag region. Γ_0 represents the contractile part of the pole region, which allows the operating length of the spindle to be reset at different levels, using the gamma motorneural pathways. Assuming a constant Γ_0, this parameter will not play a role any more in further calculations. With this consideration we get the following equations:

$$M_s = k_{ss}(\Theta - \Theta_2) \quad \text{Eq.6}$$

and

$$M_s = B_s \dot{\Theta}_2 + k_{sp}\Theta_2 \quad \text{Eq.7}$$

Keep in mind, that there is a finite delay before the feedback is closed. This delay T_d includes all lags involved. Eliminating the intermediate variables, M_s and θ, we obtain the following equation for the feedback portion of the stretch reflex model:

$$M_0(t) + \frac{M_0(t)}{\tau} = \beta\left(\dot{\Theta}(t - T_d) + \frac{\Theta(t - T_d)}{\eta\tau}\right) \quad \text{Eq.8}$$

where

$$\tau = \frac{B_s}{k_{ss} + k_{sp}}$$ Eq.9

and

$$\eta = \frac{k_{ss} + k_{sp}}{k_{sp}}$$ Eq.10

Taking the Laplace transforms, we obtain the following equations:

$$\Theta(s) = \frac{M_x(s) - M_0(s)}{s\left(\frac{BJ}{k}s^2 + Js + B\right)}$$ Eq.11

and

$$M_0(s) = \beta \frac{\tau s + 1/\eta}{\tau s + 1} e^{-sT_d} \Theta(s)$$ Eq.12

SIMULINK Plan

A block diagram of the interacting of the plant and controlling system is given in Figure 4.

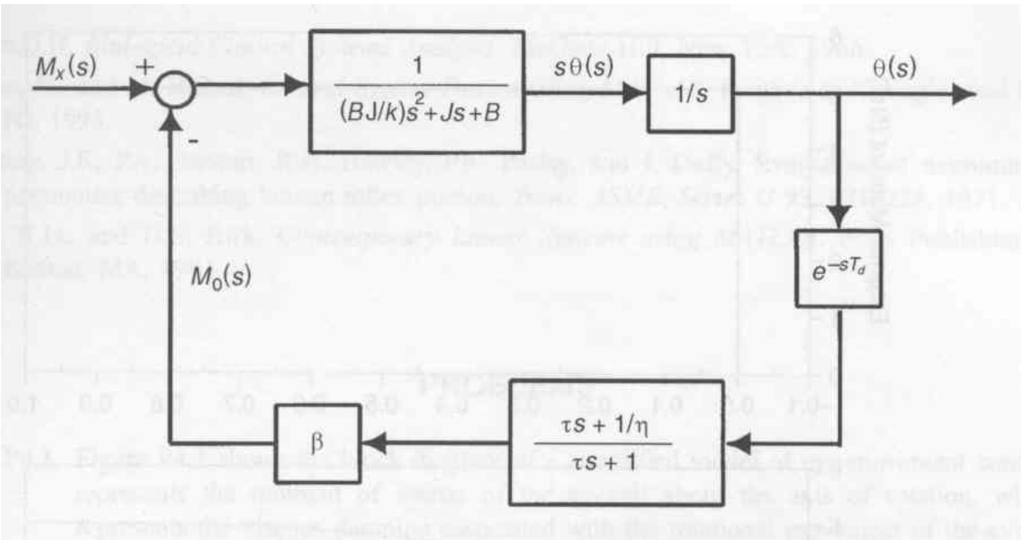

Figure 4: Block diagram of neuromuscular reflex model

The nominal parameter values used are:

J=0.1 kgm2; k=50 Nm; B=2 Nms; Td=0.02 s; τ=1/300 s; η=5; and β=100.

These values are consistent with the average physiological equivalent found in normal adult humans.

In the following Figure 5 the according Simulink plan is given:

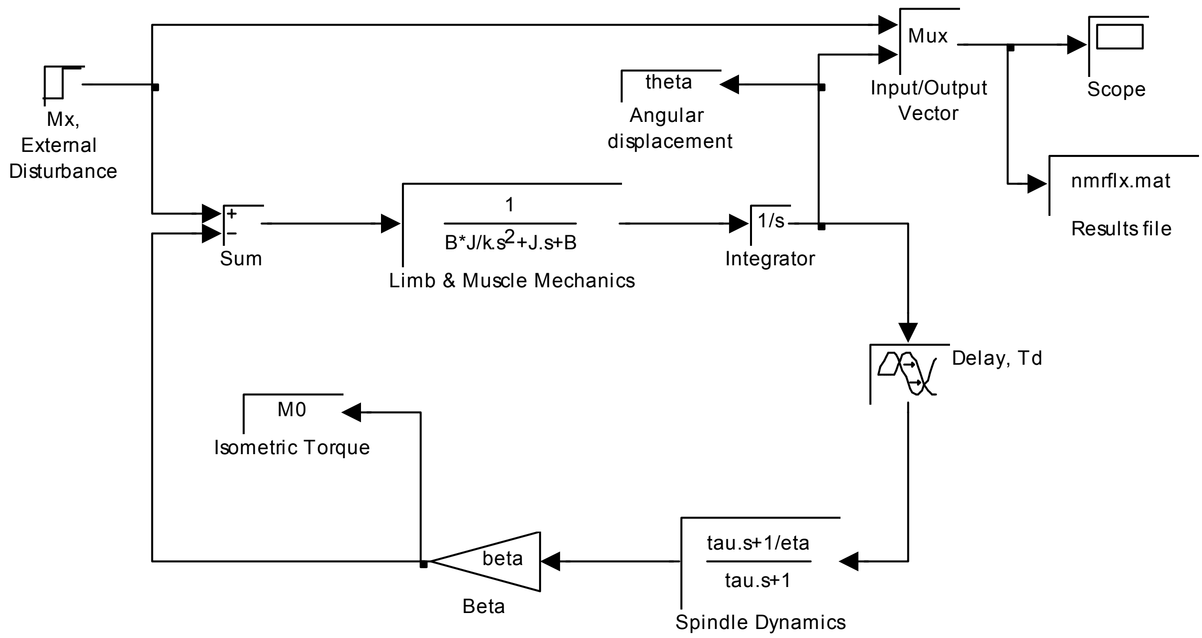

Figure 5: SIMULINK implementation of neuromuscular reflex model

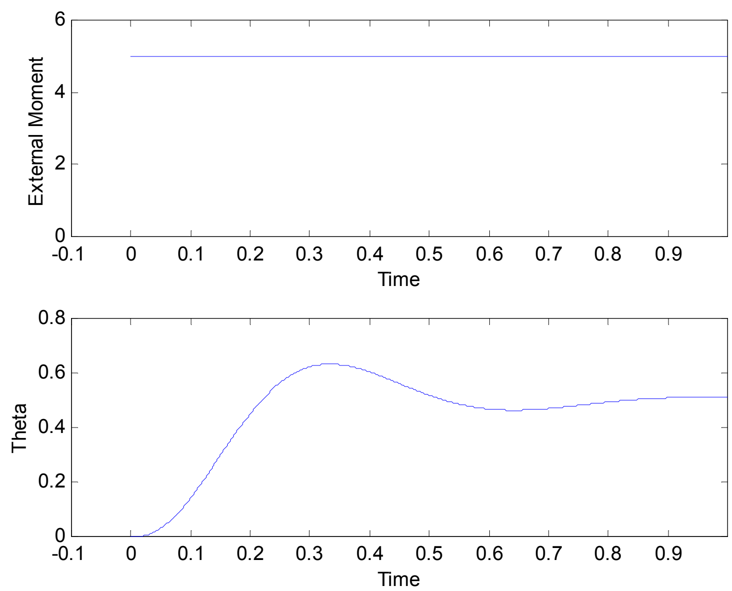

Figure 6: Sample result of simulation using Simulink